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Multiobjective Optimization Criteria for Linear Structures Subject to Random Vibrations

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Abstract: A structural optimization criterion for linear mechanical systems subject to random vibrations is presented for supporting engineer's design. It is based on a multiobjective approach whose Objective Function (OF) vector is done by stochastic reliability performance and structural cost indices. The first ones are structural reliabilities, and are evaluated for one or more failure types; they are related to designer's required performances defined in the pre-design phase. The second OF vector indices concerned cost or similar deterministic measures. The reliability based performance criteria here proposed is properly able to take into account the design required performances and so it is an efficient support for structural engineering decision making. The proposed criteria is different from other used conventional optimum design for random vibrating structure, that are based on minimizing displacement or on acceleration variance of main structure responses, but are not able to consider explicitly the required performances against structural failure.

As example of proposed criteria, the multiobjective optimum design of a Tuned Mass Damper (TMD) has been developed, for a typical seismic design problem; it deals with control of structural vibrations induced on a multi-storey building structure excited by non stationary base acceleration random process. A numerical application of this specific problem has been done with reference to a three storey building, and a sensitivity analysis is carried out. Its results are shown in a useful manner for TMD design decision support.

Key Words: Structural optimization, Multiobjective optimization, Random vibration, Tuned mass damper.

1. INTRODUCTION

Design making in structural engineering consists in applying the solution which best satisfies the required performance minimising the required recourses. Many engineers use a typical approach based on indirect or intuitive methods which depend on past experiences, subconscious motives, incomplete logical schemes, random selections and sometimes intuitive simplified mechanical schemes also. Such methods may be extremely pragmatic and applicable, but do not generally offer high optimal performance solutions, that means they are able to satisfy the given design requirement without effectively minimizing the required resources. An alternative method in structural design is based on the Optimal Structural Design (OSD), which consists in applying only logical mathematical process expressed in support of decision making. The standard Single Objective Optimization (SOO) consists in minimizing or maximising one Objective Function (OF) capable of describing system performances. In addition it may be necessary to satisfy given constraints. The OF is defined by construction and/or failure costs, total weight or one structural performance index. This alternative approach can provide at least one single optimal solution. The Multi-Objective Optimization (MOO) approach is founded on an OF vector whose dimension is greater than one, and whose elements are different structural costs and performance indices. Unlike the SOO, the MOO produces a set of possible solutions and designer must select only the possible one that better agrees with his own decisions. With reference to structural problems where dynamic loads are intrinsically random, both OF and constraints may be expressed by probabilistic entities, like covariances, spectral moments, probability of failures and similar [1]. In this field a wide class of structural engineering problems exists; it deals with structural systems thought and designed to sustain dynamic actions which can be suitably modelled as casual events rather than deterministic ones such as earthquakes, winds pressure, sea waves and rotating machinery induced vibrations. Structural responses to these actions are casual processes, so the random vibration theory is the most reliable way to assess structural answer in a probabilistic manner. Random dynamic analysis seems to be the most useful method to obtained suitable information concerning structure response and reliability (for example in [2]). In the field of structural engineering probabilistic methodologies have gained an increasing importance and now they are frequently used in order to assess structural safety problems. Probabilistic approaches can take into consideration structural parameters or loads and uncertain effects on structural answer in all cases where mechanical and excitation parameters are intrinsically random quantities. Even if this method may considerably increase the difficulties in analysis, it is the only one which can offer some essential design information that is not usually directly available by more conventional and less complicate deterministic approaches. These reasons are the result of 60 years of experiences in the field of structural dynamics; they caused the replacement of the deterministic

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approach (in which forces and structural responses are assumed as exactly known quantities) with the stochastic one, that allows to a more representative and detailed structural answer and safety evaluation. In the meantime, optimization methods have gained an increasing importance within structural design, typically based on the implicit assumption that all involved variables are deterministic. This "conventional" approach could fail when the real uncertain nature of some structural parameters is properly considered, reducing the optimal performance or at least making unfeasible the expected optimal goal. In the last decades, different approaches have been proposed using probabilistic methodologies due to computational and conceptual difficulties which properly treat uncertainty in structural optimization. The use of probabilistic methodologies has been proposed also. In standard reliability based on design optimization (RBDO) [3, 4] the objective function is minimized under probabilistic constraints instead of conventional deterministic ones. The suggested approaches for RDBO are essentially referred to time-invariant cases (see for example [5-7]). Only few contributions deal with time-variant aspects [4, 8], in which reliability is determined by the out-crossing approach and by the context of well-known FORM or SORM.

A simplified approach in structural optimization dynamic problems consists in assuming that loads are the only uncertain sources, when they have a clear un-deterministic nature as in the cases of earthquakes or wind actions: these loads are suitably modelled by stochastic processes and the standard random vibrations theory can be adopted if all the other involved quantities are assumed as deterministic. Structural response characterization is so completely described by stochastic processes with deterministic parameters. With reference to seismic engineering and seismic protection devices, a first optimum design procedure was developed by Wirshing and Campbell [9], for structures equipped with absorbers. The standard selected for the best design was an unconstrained minimization of the maximum of a suitable structural response parameter. Structural answers were obtained by statistical analysis of numerical integration of motion equations starting from generated accelerations. Constantinou and Tadjabaksh ([10-12]) developed an optimum design criterion for the seismic protection of structures with an additional first story damping device. The input was modelled by a stationary white noise Gaussian process and the adopted objective function was the system variance displacement. More recently Takewaki [13] proposed a specific and more complete stochastic approach, aimed to stiffness-damping simultaneous optimization. The sum of mean squares of response due to a stationary random excitation was minimized under constraints on total stiffness capacity and total damping capacity. An alternative interesting stochastic method for optimum design of damping devices in seismic protection was proposed by Park *et al*. [14] to minimize the total building life-cycle cost. It was based on a stochastic dynamic approach for failure probability evaluation; meanwhile the objective function was defined in a deterministic way. The standard stochastic optimization problem was also formulated by adopting the location and the amount of the viscous elastic dampers [15] or the structural shape [16] as design variables. The constraints were related to failure probability associated to the crossing of the maximum inter-storey drifts over a specific barrier level. The failure was evaluated by means of the first crossing theory in non-stationary conditions. A complete stochastically defined optimum design method is also proposed by Marano *et al*. [17], in which a reliability based optimum criterion was developed adopting a covariance approach. Both O.F. and constraints are defined in a stochastic way, where these latter impose a limit to the failure probability associated to the first threshold crossing of structural displacement over a given value. A reliability based methodology for the robust optimal design of uncertain linear structural systems subjected to stochastic dynamic loads was also presented by Papadimitriou *et al.* in [18] and [19] Safety system referred to structural displacements was used as structural performance index, under stationary white noise input conditions . The methods that authors proposed deals also with robust solution evaluating both mean and covariance of OF, by using a multiobjective optimization robust design.

Moreover all the proposed methodologies for the optimization methods of seismic devices are founded on the minimization of a single OF that quantifies the protected systems response reduction in respect to the unprotected configuration. Moreover, the OFs are expressed in terms of covariance, and their main limitation is the lack of information about final structural performance which is unknown when expressed in terms of reliability. For instance, in case of vibration protection devices the ratio between protected and unprotected structural displacement (or inertial acceleration) covariance is common used as OF. It is not at all possible to evaluate if a given required performance, commonly expressed as a limitation on maximum main system displacement or on similar response measures, is really achieved by using the protection strategy adopted even if it is possible to indicate immediately the advantages in adopting a specific seismic protection device.

For this specific reason the present work is focused on the structural optimum design standard that directly involved a performance based design (PBD) in the random vibrating structural problem. Without loss in generalities, the optimum design of a control device of vibrations is analysed as a case study regarding structures subject to seismic actions. Moreover we have to take into account that several objective functions (OFs) are involved in design decisions differently from conventional optimization (single objective function). These functions are often in conflict with each other and for them it is not possible to define a universally approved standard for "optimum" design as occurs in single objective optimization. For this reason, *Pareto dominance* and *Pareto optimality* are very important notion in MOO problems, because they are able not only to furnish a single defined optimal solution (as in SOO), but they also gives a set of possible optimal solutions satisfying, at the same time, with different performances, all designers objectives.

In this work a MOO procedure is adopted for the optimum design of seismic devices for linear structures subject to random seismic loads. This procedure adopts a bi-dimensional objective function vector, defined by using both standard deterministic cost and structural survival probability indices. An example is developed with the first OF element assumed as a deterministic device cost, and the second one is the system failure probability. The failure is defined as the first crossing out of an admissible domain of one structural response during all seismic actions; so it is the allowable top floor displacement, but other structural responses could be easily used.

The reliability evaluation is developed by using the state space covariance analysis and the Poisson hypothesis is adopted in order to evaluate the mean threshold crossing rate for the safe domain. The device seen for seismic protection is the standard Tuned Mass Damper (TMD). A single TMD located at the top of a multi degree of freedom linear system, which models a multi-storey building, is analysed. In detail the base acceleration representing seismic actions is generally modelled by a filtered white noise of a non stationary stochastic process that is able to give a quite realistic seismic loads model. In the optimization problem the design vector collects the TMD mechanical parameters i.e. frequency, the mass and damping ratio. As stated before the main innovation of the proposed approach consists in adopting the *Performance Based Seismic Design* (*PBSD*) in a MOO problem for the optimum design of a TMD in accord with modern seismic technical codes.

In general, the purpose of installing a TMD is to guarantee a suitable level of protection in the primary structure in order to assure an adequate safety level, both for the structure and its contents, towards a defined limit state. Moreover, TMD is introduced with the aim to reduce the discomfort to occupants and/or to limit the damage of equipments in particular into high rise buildings, especially when moderate (and frequent) seismic loads are taken into account in design process. This last aspect is becoming of extreme actuality in civil engineering. Recent earthquakes have in fact shown that the damage in equipments and in buildings contents can have large economic consequences. For instance, in high rise buildings, a localised damage in several acceleration sensitive non structural systems (suspend ceilings, light fixture, fire suppression piping systems, computer systems, emergency power generation systems, elevators, etc.) can affect the functionality of large portion of the building. Therefore, structural seismic design should be applied not only in order to guarantee the life safety and to prevent structural collapse, but also in order to control the damage level and the behaviour of components and systems.

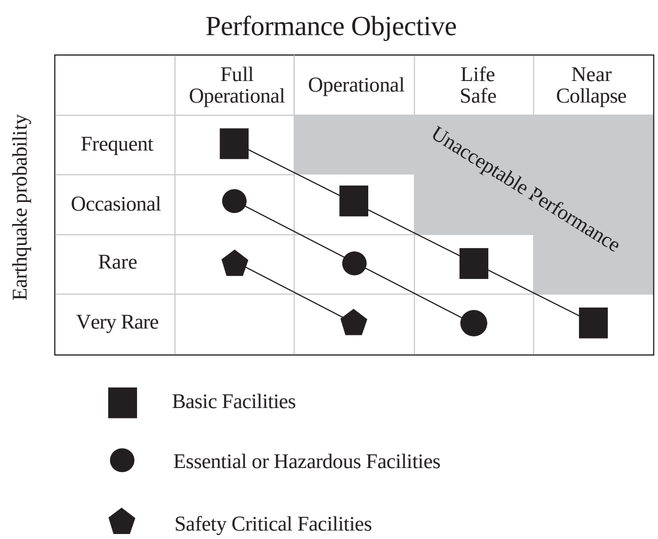
So that for a specific class of buildings, where are located strategic equipments or activities, it is fundamental guarantee, under given design earthquakes, not only structural survival (that deals with non liner structural analysis) but also specific limitation for maximum deformation, that essentially limited in a linear behaviour; this is for assurance an operatively level also after seismic events.

This concept is the base of Performance Based Seismic Design: different documents (i.e. SEAOC Vision 2000 [20, 30]) have specified in detail different performance levels required to structures.

In Table **1** one can observe that, for example, for frequent and occasional earthquakes, performance objectives are fully operational and operational. In these performance levels, the structure typically remains in the elastic range and control structure vibrations level can be efficiently obtain by adopting the TMD strategy.

The proposed approach, unlike the SOO one, is able to give pre-design information; it is extremely useful as in initial designer decisions and as the level of failure probability reduction by using a specific seismic control strategy. Using the MOO proposed in this work, the designer has the control of performances and costs in different Pareto front locations, and so he can define solution types to be adopted according to sensibilities and decisions. With more details and with reference to a TMD device, a suitable information for designer is the minimum mass ratio (that is defined as the ratio between TMD and main structural masses), necessary to increase reliability under a given level structure. This is a fundamental element for deciding if this mass ratio could be practically used or not. As an application of the proposed strategy, a multi degree of freedom system, representing a multi-storey plane frame in a simplified way, is protected by a TMD against earthquake loads. The TMD optimal solution has been obtained for different levels of admissible top floor maximum lateral displacement.

Table 1. Earthquake probability and performance objective (SEAOC Vision 2000)



2. STOCHASTIC ANALYSIS OF MULTI-DEGREE OF FREEDOM LINEAR SYSTEM SUBJECT TO RANDOM LOADS

For a generic linear *n* degrees of freedom system, excited by a forcing vector $\mathbf{f}(t)$, whose related stochastic process is the Gaussian with null mean value stochastic vector $\boldsymbol{F}(t)$, the well known differential matrix motion equation is:

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}, \mathbf{C}$ and $\mathbf{K}$ are, respectively, the deterministic mass, the damping and stiffness matrices and $\ddot{\mathbf{y}}(t)$, $\dot{\mathbf{y}}(t)$, $\mathbf{y}(t)$ consists in the acceleration, velocity and displacement vectors, whose related stochastic processes are $\ddot{\boldsymbol{Y}}(t)$, $\dot{\boldsymbol{Y}}(t)$ and $\boldsymbol{Y}(t)$.

The motion equation (1) can be written as a first order differential matrix equation, by introducing the space state vector $\mathbf{z}(t)$:

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{f}(t) \tag{2}$$

where:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix};\ \mathbf{z}(t) = \begin{Bmatrix} \mathbf{y}(t) \\ \dot{\mathbf{y}}(t) \end{Bmatrix};$$
$$\mathbf{f}(t) = \begin{Bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{f}(t) \end{Bmatrix} \tag{3}$$

The covariance matrix $\mathbf{R}(t)$ related to the space state stochastic process $\mathbf{Z}(t)$ is:

$$\mathbf{R}(t) = \langle \mathbf{z}(t)\mathbf{z}^T(t) \rangle = \begin{pmatrix} \langle \mathbf{y}(t)\mathbf{y}^T(t) \rangle & \langle \mathbf{y}(t)\dot{\mathbf{y}}^T(t) \rangle \\ \langle \dot{\mathbf{y}}(t)\mathbf{y}^T(t) \rangle & \langle \dot{\mathbf{y}}(t)\dot{\mathbf{y}}^T(t) \rangle \end{pmatrix} \tag{4}$$

And it can be evaluated by means of the *Lyapunov Covariance Matrix Equation* (see for example [21, 28]), whose the expression is:

$$\dot{\mathbf{R}}(t) = \mathbf{A}\mathbf{R}(t) + \mathbf{R}(t)\mathbf{A}^T + \mathbf{B}(t) \tag{5}$$

where $\mathbf{B}(t) = \langle \hat{\mathbf{f}}\mathbf{z}^T \rangle + \langle \mathbf{z}\hat{\mathbf{f}}^T \rangle$.

Furthermore, with the aim of finding the best design under a given stochastic load process, the n_b elements design vector **b** will be introduced to determine shape, sections or other mechanical structural features and, hence, to produce the actual value of the stiffness, mass and damping matrices**.**

With reference to define a PBD index in a stochastic way, the mechanical safety or *reliability* $r(T)$ at time T is a natural solution. It is defined as the failure survival probability, in which the failure is a partial or total damage in the interval $[0,T]$. With reference to a variety of interpretations (generally not only of a mechanical nature) of this condition, it is obvious that the definition of failure plays a central role in the reliability evaluation. Usually, the collapse can be associated to the threshold crossing probability and it is more precisely determined by the first time crossing of a structural response parameter *s(t)* through a given threshold value β. Due to the essential casual nature of actions, a random dynamic analysis is necessary and the natural way of testing the structural integrity is to evaluate the probability whether or not a structure may have a failure during its lifetime. Then, mechanical safety or reliability r at a fixed time T is defined as a failure survival probability, where the failure is a partial or total static damage in a given time $[0,T]$. So it is clear that the failure definition is very important for reliability evaluation. Normally, only two different kinds of mechanical failure are considered: the fatigue failure, due to cumulative damage, and the threshold crossing one, determined by the first time crossing of a *structural response parameter* s, through a given threshold value β. This study deals with threshold crossing failure. Regarding a generic mechanical system subject to a stochastic process, the reliability $r(T)$ defined as the survival probability when exceeding the given threshold value β, under the assumption that the initial survival probability is equal to one $(r(0)=1)$, is given by

$$r(\bar{b},T) = \exp\left\{\int_0^T h(\beta,\mathbf{b},t)dt\right\} \tag{6}$$

The hazard function $h(\beta,t)$ is represented as the probability of having a threshold crossing, in a unit time and in the absence of previous threshold crossing. Its exact formulation is still an open question. For a single degree of freedom system in the Rice's original formulation [22], the hazard function has the following general form:

$$h(\beta,\bar{b},t) = \int_0^{+\infty} \dot{s}p_{S\dot{S}}(\beta,\mathbf{b},\dot{s} \mid Q(t))d\dot{s} \tag{7}$$

where $p_{S\dot{S}}(s,\dot{s} \mid Q(t))$ is the joint probability density of $S(t)$ and $\dot{S}(t)$ processes, $Q(t)$ is the condition of excursions absent from the fixed barrier β before the time t. Difficulties related to the determination of this joint probability often impose the use of approximate solutions, one of the most commonly used is the replacement of the conditional probability of failure with the unconditional probability $p_{S\dot{S}}(s,\dot{s})$. By means of this assumption, the hazard function $h(\beta,t)$ is replaced by the threshold crossing rate $v_S^+(\beta,t)$, that depends on vector $\begin{pmatrix}\bar{S} & \dot{\bar{S}}\end{pmatrix}^T$ covariance matrix elements in approximate form. Regarding the vector of response functions of interest $\bar{S}(t)$ in reliability evaluation, the general form to express the function of the state space vector $\bar{Z}(t)$ by the linear transformation $\bar{S}(t) = \mathbf{T}_1(\mathbf{b})\bar{X}(\mathbf{b},t)$, $\dot{\bar{S}}(t) = \mathbf{T}_2(\mathbf{b})\bar{Z}(\mathbf{b},t)$ is:

$$\bar{Y}(t) = \begin{pmatrix}\bar{S} & \dot{\bar{S}}\end{pmatrix}^T = \hat{\mathbf{T}}(\mathbf{b})\bar{Z}(\mathbf{b},t) \tag{8}$$

where

$$\hat{\mathbf{T}}(\mathbf{b}) = \begin{pmatrix} \mathbf{T}_1(\mathbf{b}) & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_2(\mathbf{b}) \end{pmatrix} \tag{9}$$

and the covariance matrix in *structural response parameters* space state $\bar{Z}_S$ is

$$\mathbf{R}_{Z_S Z_S}(\mathbf{b},t) = \hat{\mathbf{T}}(\mathbf{b})\mathbf{R}_{ZZ}(\mathbf{b},t)\hat{\mathbf{T}}^T(\mathbf{b})$$

3. OPTIMIZATION CRITERIA

In all engineering fields, designers attempt to find solutions that conjugate performance and satisfaction of several requirements. Designers can obtain the optimum within the imposed conditions by using standard optimization techniques. In the field of structural engineering, structures designed in this way are safer, more reliable and less expensive than the traditional designed ones. Generally speaking, the structural optimization problem could be formulated as the selection of a set of design variables (that are the design parameters that characterize structural configuration), collected in the so above called design vector (DV) $\mathbf{b}$, over a possible admissible domain $\mathbf{\Omega}_{\mathbf{b}}$. With reference to SOO problem, the optimal DV is able to minimize a given *OF* and satisfy the assigned constraint conditions. Deterministic-based optimization is aimed to minimize structural weight or volume subject to given deterministic constraints generally referred to stresses and/or displacements. Moreover, probabilistic constraints are based on design, related to structural performance in case of reliability. Afterwards reliability theory is introduced into structural engineering and optimization with the aim of considering all existing sources of uncertainty in a more rational way. These sources can influence the structural response as well as the circumstance that the loadings applied to a structure are not exactly known. Therefore, the reliability is recognized as a performance constraint in structural engineering and so an optimum design should generally balance both cost and performance concerning structural reliability. In SOO probabilistic constraints usually define the feasible region of the design space by restricting the probability that a deterministic constraint is violated within the allowable probability of violation.

Moreover, in many real engineering problems several "efficiency" indexes (as in SOO) are involved, they could be related to structural cost or weight, structural performances and other similar standards. Each of these indices is typically conflicting with the others, and it is not possible to define an universal approved criteria of "optimum" as in SOO, where optimization is achieved by assuming that one "efficiency" index must be minimised and that the other ones must be considered as problem constraints. Moreover, this kind of choice is questionable. The above mentioned question strongly depends on designer opinion and experience. On the contrary the multiobjective optimization gives the opportunity to the designer to evaluate a set of possible solutions which satisfies more than one indices but with different performances. The definitions of these solutions are usually known as the *Pareto dominance* and *Pareto optimality* criterion, and constitute a fundamental point in the MOOPs. Regarding the Pareto optimality definition, it is assumed that a design vector $\mathbf{b}^*$ is Pareto optimal that would decrease some criterion without causing a simultaneous increase in other one criterion, if no feasible vector **b** exists. Unfortunately, this concept almost always don't give a single solution, but a set of solutions called the Pareto optimal set. The vector $\mathbf{b}^*$ corresponding to the solutions included in the Pareto optimal set are called non-dominated. Essentially, defining the generic "efficiency" index as $OF_i(\bar{b})$, a typical minimization-based MOOP is assumed as

$$\min_{\bar{b}\in\mathbf{\Omega}_{\bar{b}}}\left\{OF_1\left(\bar{b}\right),OF_2\left(\bar{b}\right),...,OF_M\left(\bar{b}\right)\right\} \tag{10}$$

Given two candidate solutions $\left\{\bar{b}_j,\bar{b}_k\right\}\in\mathbf{\Omega}_{\bar{b}}$, if:

$$\forall i\in\left\{1,...,M\right\},OF_i\left(\bar{b}_j\right)\le OF_i\left(\bar{b}_k\right)\wedge\exists i\in\left\{1,...,M\right\}: OF_i\left(\bar{b}_j\right)< OF_i\left(\bar{b}_k\right) \tag{11}$$

and it defined the two objective vectors:

$$\mathbf{v}\left(\bar{b}_j\right)=\left\{OF_1\left(\bar{b}_j\right),...,OF_M\left(\bar{b}_j\right)\right\} \tag{12}$$

$$\mathbf{v}\left(\bar{b}_k\right)=\left\{OF_1\left(\bar{b}_k\right),...,OF_M\left(\bar{b}_k\right)\right\} \tag{13}$$

where vector $\mathbf{v}\left(\bar{b}_j\right)$ is said to dominate vector $\mathbf{v}\left(\bar{b}_k\right)$ (denoted by $\mathbf{v}\left(\bar{b}_j\right)\prec\mathbf{v}\left(\bar{b}_k\right)$). Moreover, if no feasible solution $\mathbf{v}\left(\bar{b}_k\right)$ that dominates solution $\mathbf{v}\left(\bar{b}_j\right)$ exists, the $\mathbf{v}\left(\bar{b}_j\right)$ is classified as a *non-dominated* or *Pareto optimal solution*. In other terms, the candidate solution $\bar{b}_j\in\mathbf{\Omega}_{\bar{b}}$ is a *Pareto optimal solution* only and if:

$$\exists\bar{b}_k\in\grave{\mathbf{U}}_{\bar{b}}:\mathbf{v}\left(\bar{b}_k\right)\prec\mathbf{v}\left(\bar{b}_j\right) \tag{14}$$

More simply, $\bar{b}_j\in\mathbf{\Omega}_{\bar{b}}$ is a *Pareto optimal solution* if a feasible vector $\bar{b}_k\in\mathbf{\Omega}_{\bar{b}}$ which would decrease some criterion without causing a simultaneous increase in at least one other standard [23] does not exist. Unfortunately, the Pareto optimum almost always does not give a single solution but rather a set of solutions and it cannot proceed in an analytical way. The collection of all *Pareto optimal solutions* are know as the *Pareto optimal set* or *Pareto efficient set*. Instead, the corresponding objective vectors are described as the *Pareto front* or *Trade-off surface*. Normally, the decision about the "best solution" to be adopted is formulated by the so-called (human) *decision maker* (DM). The case in which the DM doesn't have any role and a generic *Pareto optimal solution* is considered acceptable (*no–preference based methods*) is extremely rare. On the other hand, several *preference–based methods* exist in literature, although this particular aspect of research tends to have been somewhat overlooked. A more general definition of the *preference–based method* considers that the preference information influences the search [23, 24]. Thus, in *a priori methods*, DM's preferences are incorporated before the search begins. Therefore, based on the DM's preferences, it is possible to avoid producing the whole *Pareto optimal set*. In *progressive methods*, the DM's preferences are incorporated during the search. This scheme offers the sure advantage of driving the search process, but the DM may be unsure of his/her preferences at the beginning of the procedure and may be informed and influenced by information that becomes available during the quest. A final classification of the methods includes the one "*a posteriori"*. In this case, the optimiser carries out the *Pareto optimal set* and the DM chooses a solution ("search first and decide later"). Many researchers view these approaches as standard so that, in the greater part of the circumstances, a MOOP is considered resolved only when all *Pareto optimal solutions* are individualized. For instance, an extremely diffused *a posteriori approach* is denominated as *Aggregating functions* in which multiple objectives are combined into a single one. In this field, the Weighed Sum Method is frequently adopted [25]. It consists in a single linear combination of individual objectives, so a scalar parameter (so-called weighting coefficient) is used with different values to define the *Pareto front*. This method, as well as others Aggregating functions techniques, are not efficient for MOOPs because they are not able to find multiple solutions in a single run and multiple runs do not guarantee the definition of the true Pareto front [26]. Moreover, in the category of *a posteriori approaches*, *Evolutionary Multi-Objective Optimization* is widely used. In [27] an algorithm is proposed to find constrained Pareto-optimal solutions based on the characteristics of a biological immune system (Constrained Multi-Objective Immune Algorithm, CMOIA). In the field of EMOO, the most adopted algorithms are the Multiple Objective Genetic Algorithm (MOGA) [28] and the Nondominated Sorting in Genetic Algorithm (NSGA) [29].

In this case, the MOOP is defined by:

$$\textit{find} \quad \mathbf{b} \in \mathbf{\Omega_b} \tag{15}$$

$$\textit{which minimizes} \quad \overline{\mathrm{OF}}(\mathbf{b}, t) \tag{16}$$

the OF vector is defined as:

$$\overline{OF}(\mathbf{b}, t) = \left\{ OF_1(\mathbf{b}, t), OF_2(\mathbf{b}, t) \right\} \tag{17}$$

$$\text{Where } OF_2(\mathbf{b}, t) = \Pr\left(G(\mathbf{S}(\mathbf{b}, \tau)) \geq 0 \mid \tau < T \right) \tag{18}$$

4. MULTIOBJECTIVE PERFORMANCE RELIABILITY OPTIMIZATION OF TMD IN SEISMIC PROTECTION

Traditional optimum design of TMD is based on protected system mean-square response minimizing (see for example [30]). In this study, a performance reliability optimum design is developed for a tuned mass damper positioned on a simple one degree of freedom linear structural system. The innovation of this approach is in considering that the optimization has to be performed by satisfying a design performance expressed in a full stochastic way by a limitation on failure probability. It is well-known that a TMD can be designed to control only a single structural model . Given the properties of the mode which needs to be controlled, the problem is essentially the same as designing a TMD for a SDOF structure. Therefore, structure is described by means of a single degree of freedom system and is equipped with a linear single tuned mass damper with the aim of reducing undesirable vibrations levels caused by dynamic loads acting at its foundation, and here modelled by means of a general filtered white noise stationary stochastic process.

In order to improve TMD efficiency it is imperative to define the optimum mechanical parameters (i.e. the optimum tuning frequency, damping and mass ratio) of TMD. Although the basic design concept of TMD is quite simple, the parameters of TMD system must be obtained through an optimal design procedure in order to satisfy performance requirements. For these reasons, the determination of optimal design parameters of TMD has become very crucial to enhance the control effectiveness.

A performance reliability based optimization is adopted to carry out an optimum TMD design. More precisely, a minimum of the mass ratio, that is the ratio of the added mass on the one of the structure, is investigated, together with the minimization of a performance index on structural reliability. The choice of the mass ratio as function to be minimized depends on the fact that this quantity can be strictly related to the total cost of the vibration control device. In general it is evident that, the limitation of the TMD mass is a primary necessity of the designer, both in mechanical and in civil engineering. Of course the increase of TMD mass will raise location volume, total structure vertical dead load, and will grow in stiffness and damping connections and similar, so a primary strategy in TMD design is to evaluate the minimum mass that this device needs to satisfy the given required performances. Moreover, this aspect is directly related to the circumstance that in the usual range of mass ratio between TMD and primary structure, by increasing this parameter, vibrations control efficiency will augment. This tendency is not strictly monotonic, because a minimum exists, and corresponds to the optimal mass ratio, but the value is usually too greater to be realistically and economically applicable in applied engineering [17].

The second OF vector element which is expressed as a structure reliability performance index is related to the failure probability associated to the crossing of the protected system displacement over an allowable limit, and is a function of designer decision. The minimum reliability level utilized in order to define the constraint is also assigned according to designer's decisions which depend on the risk level assumed as being acceptable for each given condition.

5. STATE SPACE MODEL OF THE SYSTEM

A standard way in modelling *TMD* is by a mass-dashpot-spring system (the secondary system) attached to the top of a linear MDoF system (Fig. **1**). Its main scope is to reduce unacceptable vibrations on the main structural system and therefore on the damage level and failure probability. In this specific case, the base excitation action on the building is treated as a non stationary filtered stochastic process. It is quite important to represent the evolutionary nature of response processes, given the effect that this characteristic has on structural reliability. A simpler and less computing cost could be obtained by treating the process as a stationary one, but it could overestimate the real final reliability, so that the engineering decision based on the optimization criteria could be strongly diverse from the real physical phenomenon. Therefore, a time modulated input process is here adopted for base acceleration description, and the system motion equations in Fig. **1** are:

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = -\mathbf{M}\mathbf{r}\ddot{X}_b \tag{19}$$

where $\mathbf{M}, \mathbf{C}$ and $\mathbf{K}$ are, respectively, the deterministic mass, damping and stiffness (n+1)x(n+1) matrices. The

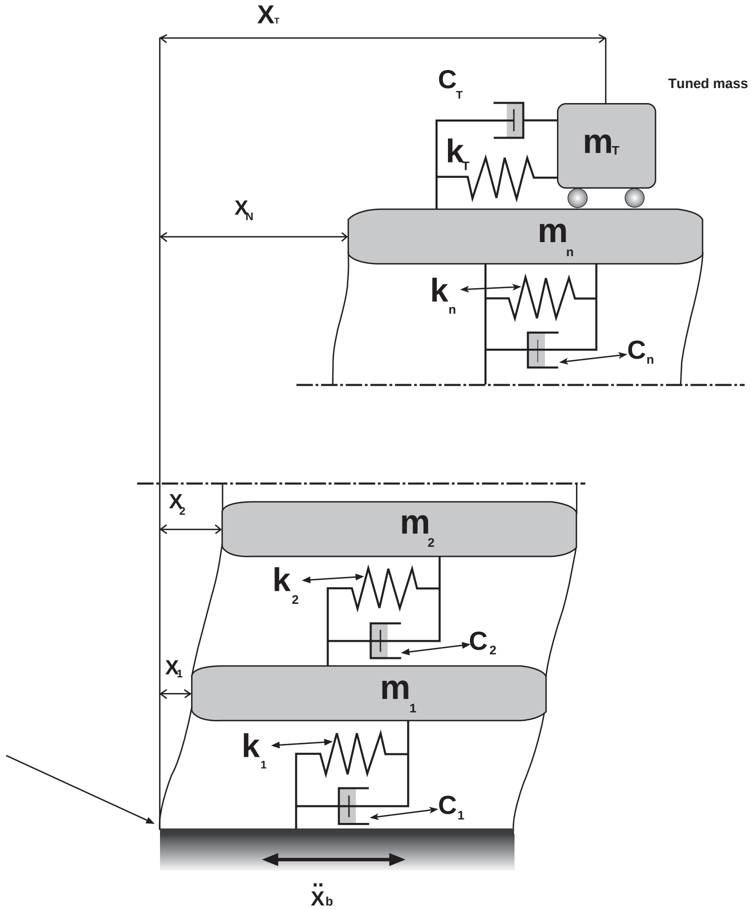


Fig. (1). Schematic model of a MDoF structural system equipped with a TMD.

(n+1) vectors $\mathbf{X} = \left(x_1\ x_2.....x_n\ x_T\right)^T$, $\dot{\mathbf{X}} = \left(\dot{x}_1\ \dot{x}_2.....\dot{x}_n\ \dot{x}_T\right)^T$ and $\ddot{\mathbf{X}} = \left(\ddot{x}_1\ \ddot{x}_2.....\ddot{x}_n\ \ddot{x}_T\right)^T$ collect the displacements, velocities and accelerations of *n* floors and of the *TMD* relative to the ground, and finally $\mathbf{r} = \left(1...1\right)^T$.

The *TMD* mechanical characteristics are described by parameters m_T, k_T and c_T, respectively, the mass, the stiffness and the damping of the *TMD.*

By adding the filter motion equation in equation (19) we obtain:

$$\ddot{\mathbf{X}}(t) = -\left[\mathbf{M}^{-1}\mathbf{C}\right]\dot{\mathbf{X}} - \left[\mathbf{M}^{-1}\mathbf{K}\right]\mathbf{X} + \mathbf{r}\\ (2\xi_f\omega_f X_f + \omega_f^2 X_f) \tag{20}$$

Introducing the space state vector

$$\mathbf{Z} = \left(\mathbf{X}\ \ X_f\ \dot{\mathbf{X}}\ \ \dot{X}_f\right)^T, \tag{21}$$

in the state space, equation (20) becomes

$$\dot{\mathbf{Z}} = \mathbf{A}\mathbf{Z} + \mathbf{F} \tag{22}$$

where the system matrix $\mathbf{A}$ is

$$\mathbf{A} = \begin{bmatrix} 0^{(n+2)(n+2)} & I^{(n+2)(n+2)} \\ -H_K & -H_C \end{bmatrix} \tag{23}$$

and the two sub-matrices $(n+2)(n+2)$ $\mathbf{H}_K$ and $\mathbf{H}_C$ are (:)

$$\mathbf{H}_K = \left[\begin{array}{c|c} & \omega_f^2 \\ (\mathbf{M}^{-1}\mathbf{K})^{(n+1)(n+1)} & \omega_f^2 \\ & ... \\ & \omega_f^2 \\ \hline 0\ \ ..\ \ ..\ \ 0 & -\omega_f^2 \end{array}\right] \tag{24}$$

$$\mathbf{H}_C = \left[\begin{array}{c|c} & 2\xi_f\omega_f \\ (\mathbf{M}^{-1}\mathbf{C})^{(n+1)(n+1)} & 2\xi_f\omega_f \\ & ... \\ & 2\xi_f\omega_f \\ \hline 0\ \ ..\ \ ..\ \ 0 & -2\xi_f\omega_f \end{array}\right] \tag{25}$$

In this work the non-stationary Kanai-Tajimi (K-T) stochastic seismic model [31] is used to describe the earthquake ground acceleration. This model has found wide application in random vibration analysis of structures because it provides a simple way to describe ground motions characterized by a single dominant frequency. The model is obtained by the use of a simple filtered white noise linear oscillator, which, in its original formulation, treats earthquakes as stationary random processes. However, accelerograms clearly show their strongly non-stationary nature both in amplitude and frequency contents, so a generalized no stationary K-T model is given by enveloping the stationary input stochastic process (in this case a stationary Gaussian white noise process $w(t)$ which is supposed to be generated at the bed rock) through a deterministic temporal modulation function $\varphi(t)$ which controls the time amplitude variation without affecting the earthquake frequency content.

Following the above considerations, the total acceleration $\overline{\ddot{X}}_b(t)$, acting at the base of the structure, is given by summing the contribute of inertial force $\ddot{X}_f(t)$ of the K-T filter and the time-modulated white noise excitation $\varphi(t)w(t)$, as follows

$$\begin{cases} \overline{\ddot{X}}_b(t) = \ddot{X}_f(t) + \varphi(t)w(t) \\ \ddot{X}_f(t) + 2\xi_f\omega_f\dot{X}_f(t) + \omega_f^2 X_f(t) = -\varphi(t)w(t) \end{cases} \tag{26}$$

where $X_f(t)$ is the displacement response of the K-T filter, ω_f is the K-T filter natural frequency and ξ_f is the K-T filter damping coefficient.

Regarding the modulation function $\varphi(t)$, different formulations have been proposed in literature. In this paper, the one suggested by Jennings [32] is used and has the following form:

$$\varphi(t) = \begin{cases} \left(\frac{t}{t_1}\right)^2 & t < t_1 \\ 1 & t_1 \le t \le t_2 \\ e^{-\beta(t-t_2)} & t > t_2 \end{cases} \tag{27}$$

where $t_d = t_2 - t_1$ is the time interval where the peak excitation is constant. Parameters are assumed as $t_1 = 3(\text{sec})$, $t_2 = 15(\text{sec})$ and $\beta = 0.4(\text{sec}^{-1})$.

The Power Spectral Density (PSD) intensity constant S_0 can be related to the standard deviation $\sigma_{\ddot{x}_b}$ of ground acceleration by means of the following relation:

$$S_0 = \frac{2\xi_f\sigma_{\ddot{x}_b}^2}{\pi\left(1+4\xi_f^2\right)\omega_f} \tag{28}$$

and assuming $PGA = 3\sigma_{\ddot{x}_b}$, the relation between PGA and spectral density is:

$$S_0 = \frac{2\xi_f\left(PGA\right)^2}{3^2\pi\left(1+4\xi_f^2\right)\omega_f} = 0.0707\frac{\xi_f\left(PGA\right)^2}{\left(1+4\xi_f^2\right)\omega_f} \tag{29}$$

where ξ_f and ω_f are the damping ratio and pulsation frequency of the filter.

In the present study, a stochastic model is considered for a typical earthquake expected on the ground, having moderate-high flexibility to perform the sensitivity analysis. The given earthquake is characterized by an energy content concentrated in the range of 1-4 Hz with PGA equal to 0.35 (g) (value that generally represents a ground motion of high intensity). In the K-T model the values are: $\omega_f = 3\pi$ *(rad/sec)*, $\xi_f = 0.45$ and $S_0 = 175.5$ (m^2/sec^3).

The *Liapunov* equation, whose solution supplies the system response covariance has the same form of equation (5).

As stated above, a performance reliability criterion is here adopted in order to perform the optimum design of a TMD device in the protection of a general multi - storey building subject to a filtered non-stationary base acceleration input process. The structural required performance concerns structure reliability associated to the maximum lateral building displacement. The possibility to satisfy a demanded limitation of maximum lateral displacement has been investigated with a TMD device placed at the top storey of the building whose cost has to be limited by minimising its mass.

The optimum design is aimed to define TMD mechanical characteristics which are the frequency ω_T and the damping ratio ; they are collected in the design vector $\mathbf{b} = \left[\omega_T, \xi_T\right]^T$.

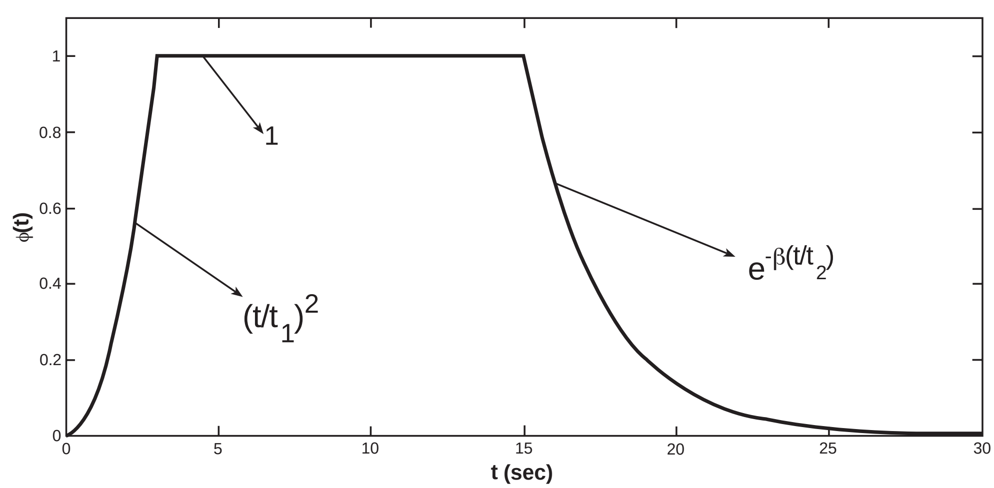


Fig. (2). Jenning's modulating function.

Indicating with γ_m the mass ratio, it is defined as the TMD mass respect to the total building:

$$\gamma_m = \frac{m_{TMD}}{\sum_{i=1}^{n_f} m_i} \tag{30}$$

where n_f is the total floors number and m_i is the mass of each storey.

A possible strategy that could be adopted for the structural optimization of TMD mechanical parameters is the minimization of γ_m and of the system failure probability, here related to the crossing of the top storey lateral displacement over a fixed allowable value. In this case, indicating with $P_f\left(\mathbf{b}, x_{adm}, T\right)$ the structure failure probability at time T (the end of structural vibrations), it is assumed that the conventional structural failure takes place when the building top storey lateral displacement x_n crosses a fixed threshold value x_{adm}. This performance index (or its complementary reliability $r(\mathbf{b}, x_{adm}, T) = 1 - P_f$) must be evaluated, with respect to the first exceeding of a threshold value x_{adm}. At the beginning of the seismic action, keeping in mind the assumption that $r(\mathbf{b}, x_{adm}, 0) = 1$n, the approximate Poisson formulation for a symmetric barrier gives:

$$P_f(\mathbf{b}, x_{adm}, T) = 1 - \mathrm{e}^{-2\int_0^T \nu^+(\mathbf{b}, x_{adm}, \tau) d\tau} \tag{31}$$

where, assuming that the above stochastic processes are Gaussian with null mean values (see for example [14, 33]0), the threshold crossing rate $\nu^+(\mathbf{b}, x_{adm}, t)$ is :

$$\nu_S^+(\mathbf{b}, x_{adm}, t) = \frac{1}{2\pi} a^{(1)}(\mathbf{b}, t) a^{(2)}(\mathbf{b}, t) a^{(3)}(\mathbf{b}, x_{adm}, t) \chi \left[d_S(\mathbf{b}, x_{adm}, t)\right] \tag{32}$$

where

$$a^{(1)}(\mathbf{b}, t) = \frac{\sigma_{\dot{X}_n}(\mathbf{b}, t)}{\sigma_{X_n}(\mathbf{b}, t)} \tag{33}$$

$$a^{(2)}(\mathbf{b}, t) = \sqrt{1 - \rho^2_{X_n \dot{X}_n}(\mathbf{b}, t)} \tag{34}$$

$$a^{(3)}(\mathbf{b}, x_{adm}, t) = \exp\left\{-\frac{1}{2}\left(\frac{x_{adm}}{\sigma_{X_n}(\mathbf{b}, t)}\right)^2\right\} \tag{35}$$

$$\chi\left[d_S(\mathbf{b}, x_{adm}, t)\right] = \exp\left(-\frac{d_X(\mathbf{b}, x_{adm}, t)^2}{2}\right) + d_X(\mathbf{b}, x_{adm}, t) \sqrt{2\pi}\left[1 - \Phi\left\{d_X(\mathbf{b}, x_{adm}, t)\right\}\right] \tag{36}$$

$$d_X(\mathbf{b}, x_{adm}, t) = \frac{x_{adm}}{\sigma_{X_n}(\mathbf{b}, t)}\left(\frac{\rho_{X_n \dot{X}_n}(\mathbf{b}, t)}{\sqrt{1 - \rho^2_{X_n \dot{X}_n}(\mathbf{b}, t)}}\right) \tag{37}$$

$\sigma_{X_n}(\mathbf{b}, t)$ and $\sigma_{\dot{X}_n}(\mathbf{b}, t)$ are the standard deviations of $X_n(\mathbf{b}, t)$ and $\dot{X}_n(\mathbf{b}, t)$ and $\rho_{X_n \dot{X}_n}(\mathbf{b}, t)$ is their correlation factor.

Hence, the MOO problem is defined by collecting in an OF vector both the deterministic cost index and the reliability measure(,) so that the multiobjective optimal criteria could be stated as:

$$\textit{find} \qquad \mathbf{b} \in \grave{\mathbf{U}}_d \tag{38}$$

$$\textit{which minimizes} \quad \overline{OF}\left(\mathbf{b}, T\right) = \left\{\gamma_m, P_f\left(\mathbf{b}, T\right)\right\} \tag{39}$$

obtaining a numerical problem that can be solved with above mentioned methods. Due to the relatively regular solution of the problem, the standard weight method has been here adopted in the Pareto optimal set definition.

6. NUMERICAL RESULTS

A numerical application of proposed optimization criteria for TMD is developed in this section. It deals with a three floors building model subject to nonstationary process that models real earthquake phenomena. Mechanical characteristics, regarding the storey masses and the lateral stiffness of the main structure with three degrees of freedom are reported in Table **2**. The damping matrix is assumed as proportional by using the first and second natural frequencies. Modal system natural periods are reported in Fig **3**.

As above stated the optimum design of TMD is aimed to evaluate the design vector $\mathbf{b} = \left[\omega_T, \xi_T\right]^T$ which simultaneously minimizes the performances expressed by , which requires that system failure must be lower than a given limit, depending on the specific violated limit state. A Pareto optimum front is obtained by solving the original problem.

The plot of the objective functions whose non-dominated vectors are in the Pareto optimal set is called the *Pareto front*. It has been obtained, for the present example, by obtaining the minimum probability of failure for different mass ratio values in a given interval of interest from the engineering point of view. It is well known that only limited or small values of this parameters are realistically utilizable in civil engineering TMD applications. By using this way, a standard single OF optimization criteria has been used to get Pareto front points. In more details, a Matlab genetic algorithm has been used for this goal.

Table. 2

System Parameters	First Floor	Second Floor	Third Floor	
Mass	$5 \bullet 10^6$	$4 \bullet 10^6$	$3 \bullet 10^6$	(kg)
Stiffness	$6 \bullet 10^3$	$6 \bullet 10^3$	$4.2 \bullet 10^3$	(N/m)

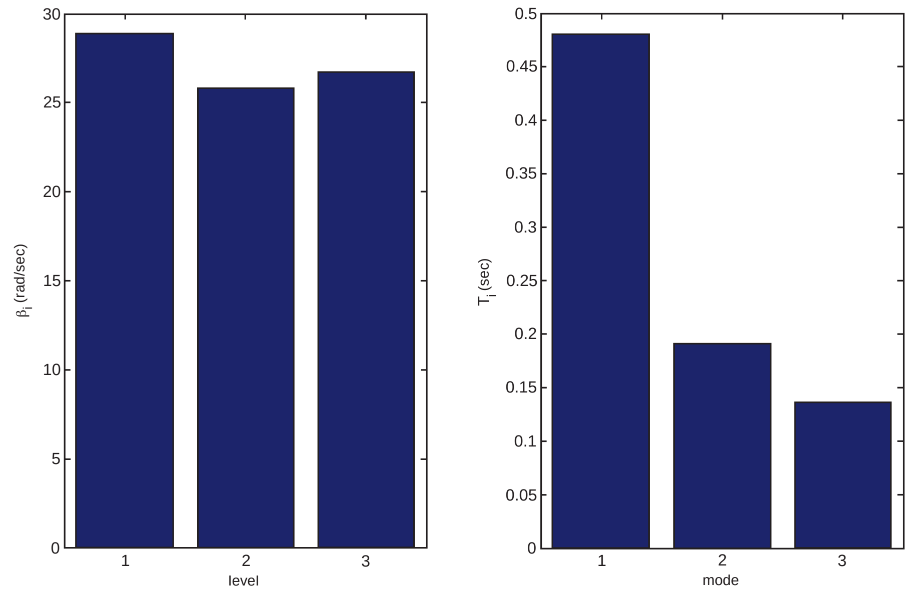


Fig. (3). square roots of storey ratio between lateral stiffness and masses $\beta_i = \sqrt{\frac{k_i}{m_i}}$ (left) and modal natural period of analysed structure (right).

Fig. **4** shows the Pareto fronts. The minimized mass ratio γ_m is plotted on the x-axis and corresponds by Pareto standard to the minimum of failure probability P_f , plotted on the y-axis. The other optimum TMD parameters, ω_T and ξ_T , are respectively plotted in Figs. **5** and **6**. Different coloured lines in Figs. (**4**), (**5**) and (**6**) correspond to the various admissible structural roof displacements x_{adm} which correspond to 8 and 9 cm. Therefore the obtained results can be adopted to develop a performance – reliability based optimum design of TMD. The optimum solution **b** is more precisely obtained , with a limit $\tilde{P}_f$; it minimizes γ_m and satisfies the required performance (for example the horizontal line for $\tilde{P}_f is 10^{-2}$ which corresponds to $r_{\min} = 0.99$). It can be observed that in terms of reliability a higher performance level in terms of reliability requires a higher mass ratio. Moreover, it is possible to deduce that for some values of maximum roof displacement, the optimality cannot be reached by using a TMD. This means that a solution does not always exist. On the other hand, in some cases, the required performance is attained without using the TMD vibrations control strategy. It is obvious that different results depend on the particular values of $\tilde{P}_f$ and x_{adm} . Therefore, the proposed method represents a useful support for designer decisions, offering a complete and clear scenario of all possible solutions regarding both limit displacements and required reliability.

The optimum design of the values of vector elements are reported in Figs. **5** and **6**. In the first figure the optimum TMD frequency ratio is reported on the y-axis, i.e. the ratio of the optimum TMD frequency ω_{TMD}^{opt} with respect to the structural frequency ω_s . The x-axis gives the optimum mass ratio μ . In the second figure, the optimum TMD damping ratio ξ_{TMD}^{opt} is plotted on the y-axis, whereas the x-axis furnishes the optimum mass ratio γ_m .

In these two figures it can be deduced that all optimum solutions depend only on the mass ratio and not on the admissible displacement x_{adm} . This result is quite reasonable giving that the optimum solution is essentially related to the mass ratio, and in any case it tends to find the couple of optimal TMD mechanical parameters able to maximize the vibrations reduction. On the contrary, the failure level for a given mass ratio depends passively on the admissible displacement only, so that the optimal solution is not directly related to failure probability. This is a quite important results, because it implies that optimal solutions in terms of TMD parameters are independent from the system failure probability and mass ratio adopted. On the contrary it depends directly only on admissible displacement and adopted mass ratio, ones that the optimal ω_{TMD}^{opt} and ξ_{TMD}^{opt} has been defined.

Moreover, by observing these figures, it is possible to notice that when γ_m grows, two different trends can be noticed for the DV elements. Firstly, the optimum TMD frequency ratio (i.e. the ratio of the optimum TMD frequency ω_{TMD}^{opt} respect to the structural frequency ω_s), decreases to a low similar to a linear one. It starts from a value quite close to 0.95 (TMD is nearly about in resonance with the main structure) and reaches values of about 0.70. On the contrary, the optimum TMD damping ratio ξ_{TMD}^{opt} grows up as the mass ratio increases, from 0.07 up to about 0.21 (for γ_m =0.10), with an approximately parabolic law.

CONCLUSIONS

In this work has been focused a reliability performance based optimum design criteria, with reference to linear elastic multi-degree of freedom when they are structures subject to random loads. Unlike other design methods, that are founded on minimizing system mean-square response, in this proposed criteria, a reliability-based performance index is considered, with the aim of be more useful and efficient in supporting design engineering decisions. This approach has been adopted for defining a MOO criteria also founded on system performance reliability. The optimum design of mechanical characteristics of TMD has been analysed as a case of study. The criterion selected for the optimum design is based on the minimization of the mass of the vibrations control device and on a performance reliability associated to the system displacement crossing beyond a given allowable displacement.

The original one-dimensional optimum design has been transformed into a multi-dimensional criterion. Then the Pareto fronts have been obtained. The sensitivity analysis carried out by varying the admissible displacement, have shows that the optimum solution not always exists, and that in some cases the required performance is extracted without

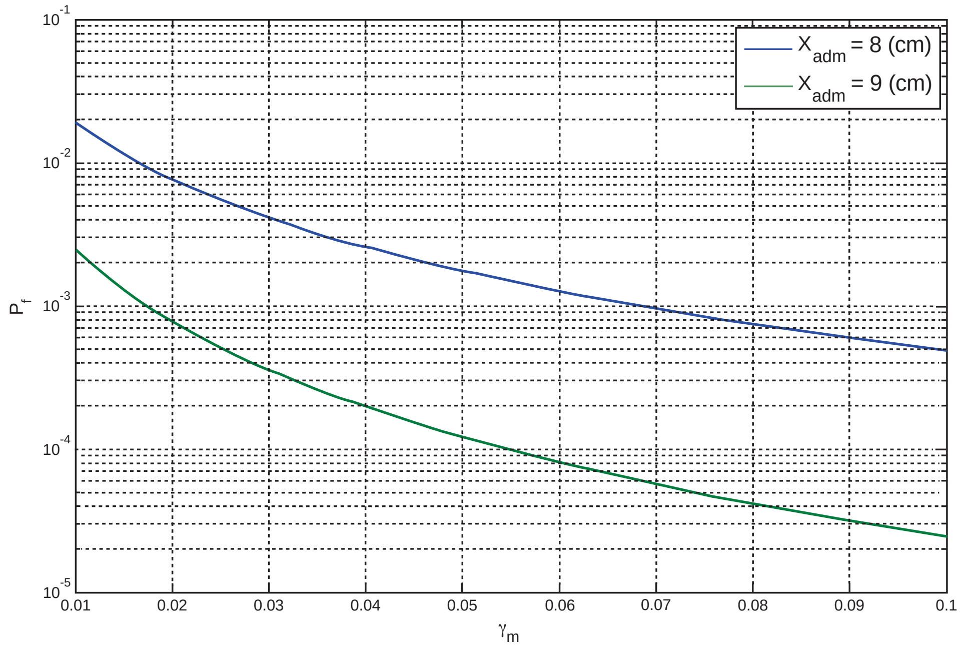


Fig. (4). Pareto fronts for different admissible displacements.

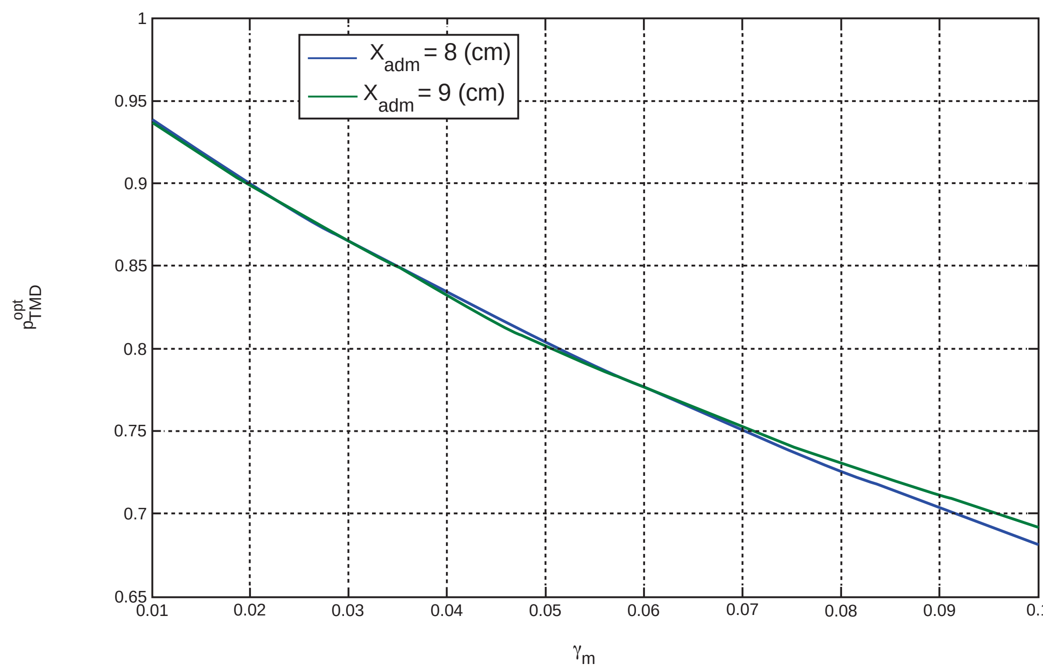


Fig. (5). Optimum TMD frequency ratio for different admissible displacements.

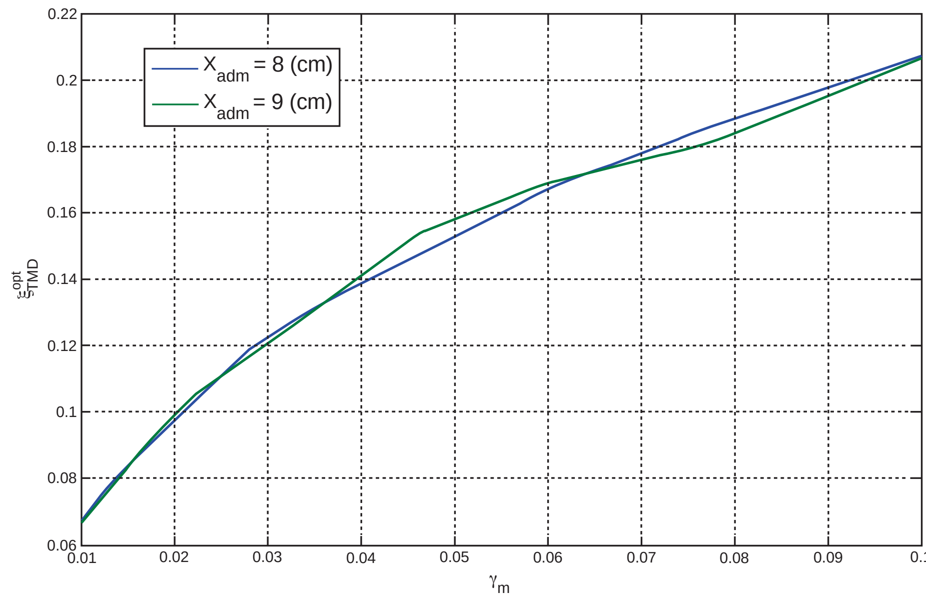


Fig. (6). Optimum TMD damping ratio for different admissible displacements.

using the TMD strategy. The results obtained can be used as a suitable decision making support for designers in evaluating the efficiency of TMD systems to obtain assigned required performances in vibrations control. It has been observed that optimal solutions of TMD parameters are independent from system failure probability and mass ratio. On the contrary, those two parameters are strongly correlated each other, ones that optimal TMD frequency and damping ratio are defined.

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