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The Open Civil Engineering Journal, 2012, *6*, 107-118

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Practical Nonlinear Analysis for Limit Design of Reinforced Masonry Walls

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Abstract: Two modeling techniques for practical nonlinear static analysis are implemented to support the development and usage of a new Limit Design method for special reinforced masonry shear walls. The new seismic design alternative is under consideration for future versions of the building code requirements for masonry structures in the U.S.

The proposed simplified models were applied to a planar one-story wall with two openings and the relevant output data from nonlinear static analyses were compared to the output from a refined computer model. Results of the comparison indicate that the proposed models were sufficiently accurate in determining the usable base-shear strength of the perforated wall.

Keywords: Displacement-based design, limit analysis, pushover, seismic design, shear walls, yield mechanism.

1. INTRODUCTION

Analytical models are presented for performing practical nonlinear static analysis of masonry shear walls proportioned and detailed to resist strong ground motions. Two simplified modeling techniques were implemented to support the development and usage of the Limit Design code provisions presented in Table **1**. These provisions are part of a new design alternative for special reinforced masonry shear walls under consideration for the 2013 Masonry code by the Masonry Standards Joint Committee (MSJC) [1]. Although the Limit Design code is written to allow limit analysis based on hand calculations using principles of virtual work, the computer models proposed here directly take into account the effects of varying axial load caused by an increase in lateral forces. The code notation and other terms used throughout this paper are defined in Appendix A.

Program SAP2000 (version 15) by Computers and Structures Inc. [2] is used to implement two modeling techniques. The models are based on the predominant use of linear-elastic area elements combined with limited number of elements having nonlinear force-displacement relationships. The Limit Design code (Table **1**) assumes plastic hinge regions occur at the interface between wall segments.

The first model, the Nonlinear Layer model, modifies the area elements at potential critical (yielding) sections with special layer definitions that account for material nonlinearity. The second model, the Nonlinear Link model, substitutes the area elements at potential critical (yielding) sections with nonlinear links. The use of links is attractive because similar types of elements are readily available in standard structural analysis software other than SAP2000. Both modeling techniques are effective in identifying the yielding wall segments and in determining the base-shear strength of a perforated wall configuration.

To perform a nonlinear static analysis of a masonry shear wall configuration using either nonlinear layers or nonlinear links, the user must first develop a linear-elastic model. The linear-elastic model is used as a reference model to obtain the design roof displacement and to determine the axial forces due to the factored loads that are consistent with the design load combination producing the design roof displacement. These axial forces are used to calculate the deformation capacity of yielding wall segments based on simple rules (see Table **1**, Section X.3).

The proposed analytical models are best described through their application to an example consisting of a one-story concrete masonry wall configuration with two openings (see Fig. **1**). The openings lead to a structure comprised of three vertical wall segments connected to three joint segments coupled by two horizontal wall segments. The ends of the vertical and horizontal wall segments identify potential hinge regions. The perforated wall configuration is assumed to have a rigid diaphragm at the roof level, located at 2'-0" (610 mm) below the top of the wall. The definitions of material properties for modeling nonlinear response are characterized by the nominal material strengths shown in Fig. (**1**).

To develop the nonlinear model, the area elements located at the interface of wall segments are replaced with either nonlinear layered area elements (case of the Nonlinear Layer model) or nonlinear links (case of the Nonlinear Link model). For the wall configuration presented in Fig. (**1**), the computer model for evaluating its linear-elastic response is shown in Fig. (**2**). The linear-elastic model uses area elements with an 8-in. (203-mm) square mesh. This level of

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Table 1. Limit Design Code Provisions and Commentary, after Lepage *et al*. [6].

Code	Commentary
X *General* — The Limit Design method shall be permitted to be applied to a line of lateral load resistance consisting of Special Reinforced Masonry Shear Walls that are designed per the Strength Design provisions of Chapter 3, except that the provisions of Section 3.3.3.5 and Section 3.3.6.5 shall not apply.	**X** *General* — This section provides alternative design provisions for special reinforced masonry shear walls subjected to in-plane seismic loading. The Limit Design method is presented as an alternative to the requirements of 3.3.3.5 and 3.3.6.5. All other sections in Chapter 3 are applicable. Limit Design is considered to be particularly useful for perforated wall configurations for which a representative yield mechanism can be determined.
X.1 *Yield mechanism* — It shall be permitted to use limit analysis to determine the controlling yield mechanism and its corresponding base-shear strength, V_{lim}, for a line of lateral load resistance, provided (a) through (d) are satisfied: (a) The relative magnitude of lateral seismic forces applied at each floor level shall correspond to the loading condition producing the maximum base shear at the line of resistance in accordance with analytical procedures permitted in Section 12.6 of ASCE 7. (b) In the investigation of potential yield mechanisms induced by seismic loading, plastic hinges shall be considered to form at the faces of joints and at the interface between masonry components and the foundation. (c) The axial forces associated with load combination 7 per Section 2.3.2 of ASCE 7 shall be used when determining the strength of plastic hinges, except that axial loads due to horizontal seismic forces are permitted to be neglected. (d) The strength assigned to plastic hinges shall be based on the nominal flexural strength, M_n, but shall not exceed the moment associated with one-half of the nominal shear strength, V_n, calculated using MSJC Section 3.3.4.1.2.	**X.1** *Yield mechanism* — This section defines the basic conditions for allowing the use of limit analysis to determine the base-shear strength of a line of resistance subjected to seismic loading. Item (a) allows the use of conventional methods of analysis permitted in ASCE 7 to determine the distribution of lateral loads. The designer should use the seismic loading condition that produces the maximum base-shear demand at the line of resistance. Item (b) allows the location of yielding regions at the interfaces between wall segments and their supporting members. Item (c) prescribes the use of the loading condition that induces the lowest axial force due to gravity loads. For wall segments loaded with axial forces below the balanced point, this loading condition gives the lowest flexural strength and therefore leads to lower mechanism strengths. Item (d) limits the flexural strength that is assigned to a plastic hinge so that the maximum shear that can be developed does not exceed one-half the shear strength of the wall segment. This stratagem effectively reduces the strength of the controlling yield mechanism involving wall segments vulnerable to shear failure. In addition to a reduction in strength there is a reduction in deformation capacity as indicated in X.3.2.
X.2 *Mechanism strength* — The yield mechanism associated with the limiting base-shear strength, V_{lim}, shall satisfy the following: $\phi V_{lim} \geq V_{ub}$ The value of ϕ assigned to the mechanism strength shall be taken as 0.8. The base-shear demand, V_{ub}, shall be determined from analytical procedures permitted in Section 12.6 of ASCE 7.	**X.2** *Mechanism strength* — Because the controlling yield mechanism is investigated using nominal strengths, an overall strength reduction factor of ϕ = 0.8 is applied to the limiting base-shear strength. For simplicity, a single value of ϕ is adopted.
X.3 *Mechanism deformation* — The deformation demand on plastic hinges shall be determined by imposing the design displacement, δ_u, at the roof level of the yield mechanism. The deformation capacity of plastic hinges shall satisfy X.3.1 to X.3.3. **X.3.1** The deformation capacity of plastic hinges shall be taken as $0.5\,\ell_w h_w \varepsilon_{mu} / c$. The value of c shall be calculated for the P_u corresponding to load combination 5 per Section 2.3.2 of ASCE 7. **X.3.2** The deformation capacity of masonry components where the plastic hinge strengths are limited by shear as specified in X.1(d), shall be taken as $h_w/400$, except that $h_w/200$ shall be used for masonry components satisfying the following requirements: (a) Transverse and longitudinal reinforcement ratios shall not be less than 0.001; (b) Spacing of transverse and longitudinal reinforcement shall not exceed the smallest of 24 in. (610 mm), ℓ_w / 2, and h_w / 2; (c) Reinforcement ending at a free edge of masonry shall terminate in a standard hook. **X.3.3** The P_u corresponding to load combination 5 of Section 2.3.2 of ASCE 7 shall not exceed a compressive stress of 0.3 $f'_m A_g$ at plastic hinges in the controlling mechanism.	**X.3** *Mechanism deformation* — This section defines the ductility checks required by the Limit Design method. The deformation demands at locations of plastic hinges are determined by imposing the calculated design roof displacement to the controlling yield mechanism. **X.3.1** The deformation capacity is calculated assuming an ultimate curvature of ε_{mu} / c acts on a plastic hinge length of $0.5\,\ell_w$ with an effective shear span of h_w. The resulting expression is similar to that used in 3.3.6.5.3(a) to determine the need for special boundary elements. The value of P_u includes earthquake effects and may be calculated using a linearly elastic model. **X.3.2** At locations where the hinge strength is assigned a value lower than the nominal flexural strength due to limitations in X.1(d), the deformation capacity is limited to h_w / 400 or h_w / 200 depending on the amount of transverse and longitudinal reinforcement. **X.3.3** The limit of 30% of f'_m is intended to ensure that all yielding components respond below the balanced point of the P-M interaction diagram.

discretization is sufficiently accurate considering that for a unit load applied at the roof level the resulting roof displacement is within 3% of the displacement calculated using a 1-in. (25.4-mm) square mesh. The 8-in. (203-mm) square mesh also allows a direct representation of the modular dimensions of the standard concrete masonry unit.

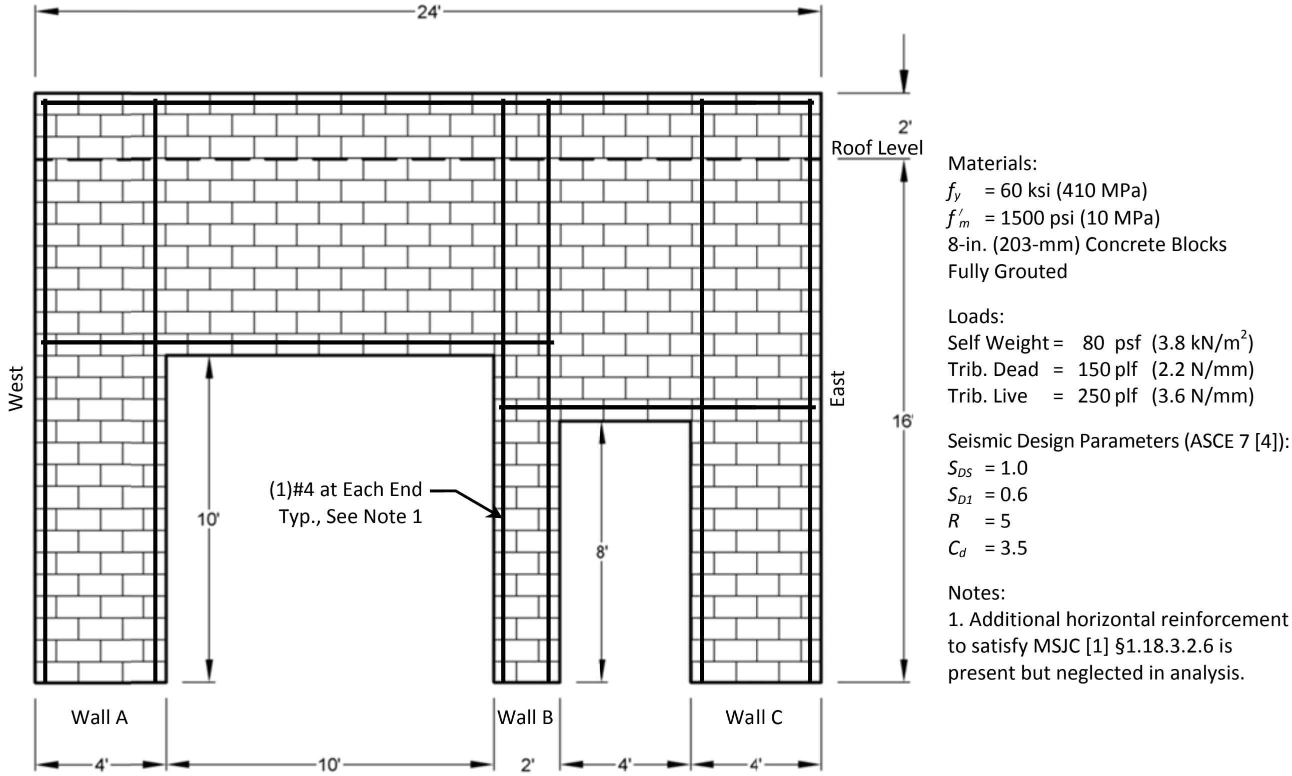

Fig. (1). Description of Wall Considered, after Lepage *et al*. [6] (1 ft = 305 mm).

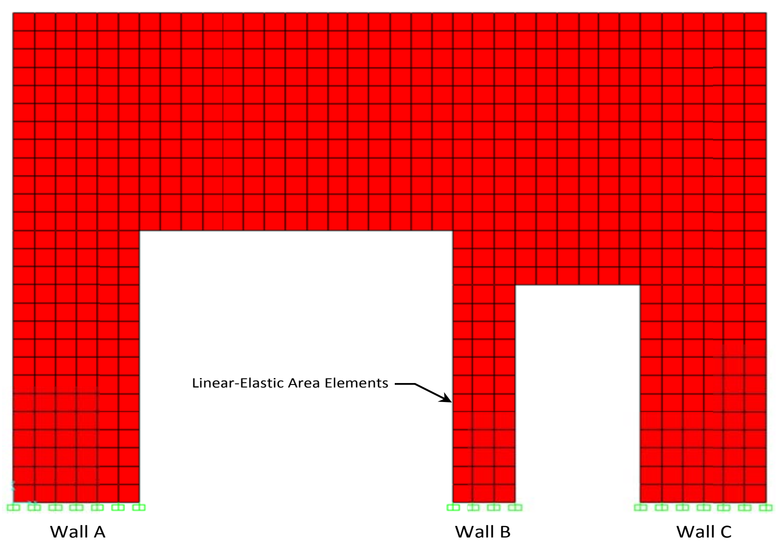

Fig. (2). Linear-Elastic Model with 8 in. by 8 in. Mesh (1 in. = 25.4 mm).

The following general assumptions and simplifications are involved in developing the 2D models presented: the structure, loads, and response are defined in one vertical plane; structural response accounts for the effects of shear, axial, and flexural deformations; the wall configuration is fixed at its base; all nodes at the roof level are constrained by a rigid diaphragm; horizontal seismic loads act at the roof diaphragm level; P-Δ effects are neglected.

To facilitate the understanding and implementation of the proposed modeling techniques, Sanchez [3] details the steps involved in creating the Nonlinear Layer and Nonlinear Link models with Program SAP2000 (version 15). It is important to emphasize that these models are not suitable for use in nonlinear dynamic analysis. Additional special definitions would be needed to properly account for the cyclic behavior involving masonry cracking or reinforcement yielding.

2. NONLINEAR LAYER MODEL

The layered shell element, available in SAP2000 (version 15), is a special type of area element that may be defined with multiple layers in the thickness direction. Each layer may represent independent materials with user-defined nonlinear stress-strain relationships. A detailed description of the advanced features of the layered shell element is presented by CSI [2].

The proposed model is based on the use of nonlinear area elements to represent the region at the interface of wall segments where yielding is likely to occur, see Fig. (**3**). The area elements outside these potential yielding regions are modeled with linear-elastic area elements using full gross section properties. For a planar wall configuration the area elements may be defined as membrane elements with layers assigned to materials with nonlinear behavior. Layers of masonry and steel reinforcement are combined to represent reinforced masonry sections. For unreinforced masonry sections, only masonry layers are used.

Material stress-strain relationships are defined to represent nonlinear axial and shear behavior of the wall segments. The in-plane flexural behavior of the walls is controlled by the axial response characteristics of the materials assigned to the layers. Independent materials are defined to represent the axial response of masonry and steel reinforcement. Masonry is assumed to have a bilinear stress-strain curve in compression and zero tensile capacity, see Fig. (**4**). Reinforcing steel is characterized by a bilinear and symmetrical stress-strain curve as shown in Fig. (**5**). The peak compressive stress of masonry is taken as 0.8 times the specified compressive strength of masonry, f'_m, and the peak stress of the reinforcing steel is based on the specified yield strength, f_y. Material property definitions neglect the strain hardening effects of steel and the expected overstrengths of steel and masonry. The nearly zero slope of the stress-strain curves at large

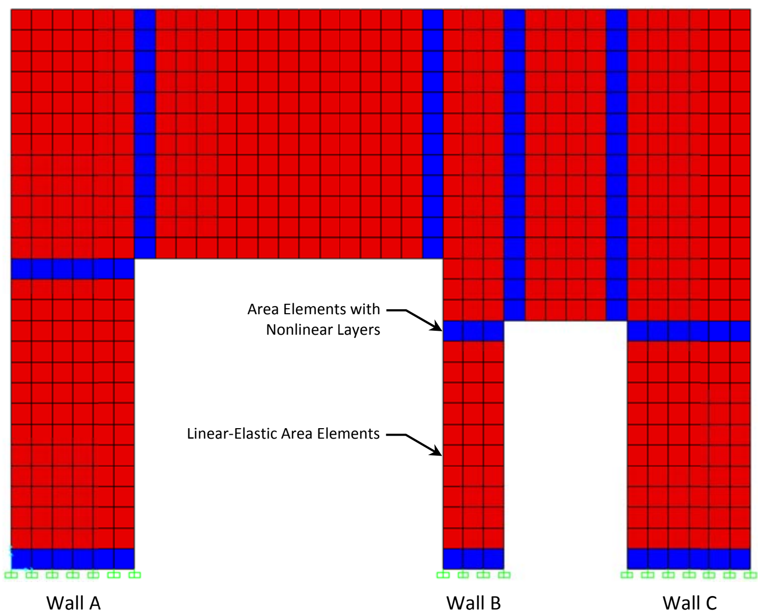

Fig. (3). Nonlinear Layer Model.

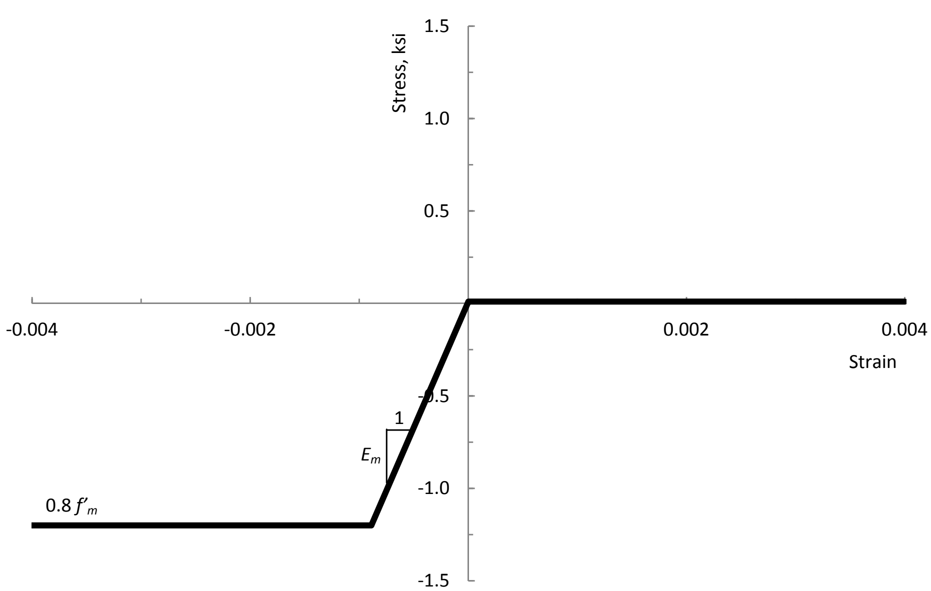

Fig. (4). Nonlinear Layer Definition, Masonry Axial Direction (1 ksi = 6.9 MPa).

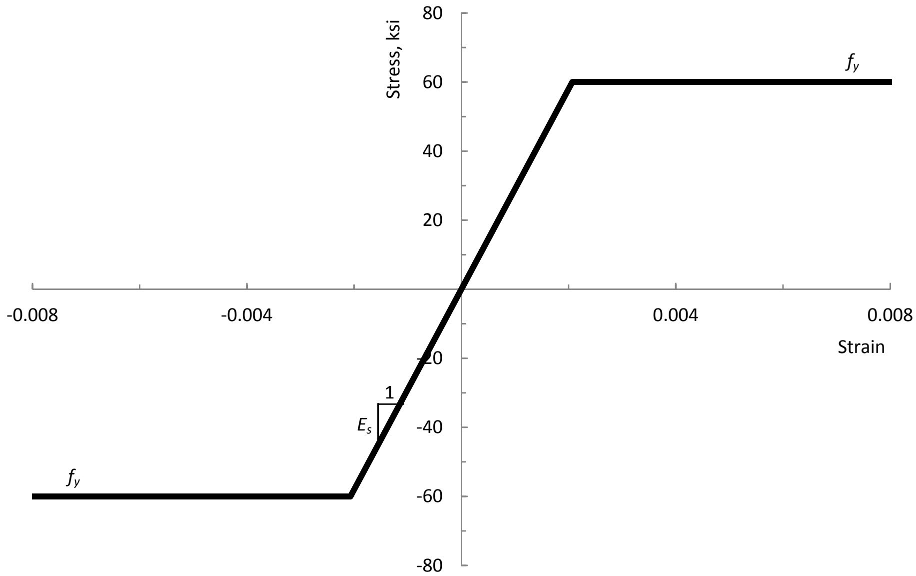

Fig. (5). Nonlinear Layer Definition, Reinforcing Steel Axial Direction (1 ksi = 6.9 MPa).

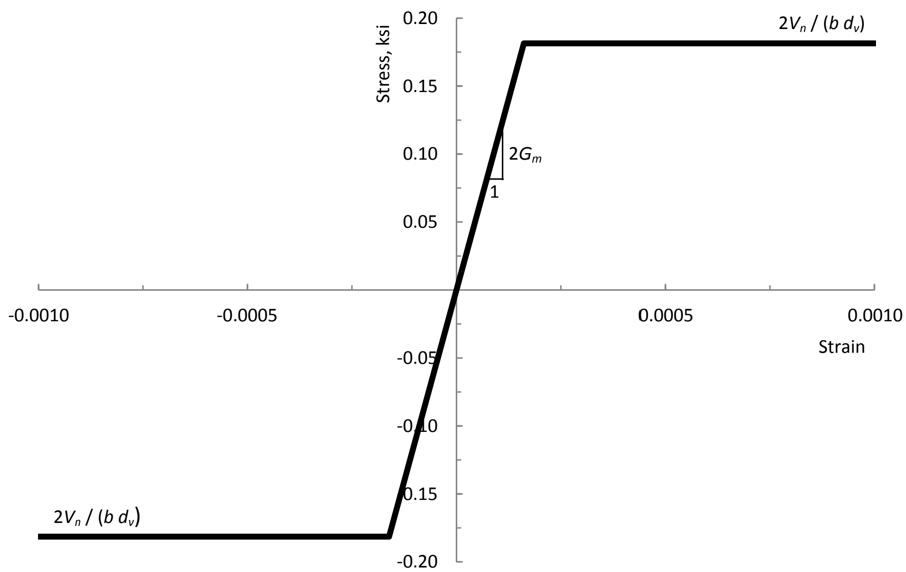

Fig. (6). Nonlinear Layer Definition, Shear Direction, Walls A and C (1 ksi = 6.9 MPa).

strains for a given direction of loading ensures a stable structure throughout the analysis. The user needs to verify that the computed output is limited to realistic usable strains.

The nonlinear shear response is also modeled using a bilinear and symmetrical stress-strain curve, see Fig. (**6**). The initial line segment of the stress-strain curve is defined by the shear modulus, G_m, taken as E_m / 2.4. The peak values in Fig. (**6**) correspond to the calculated shear strength divided by the cross-sectional area of the wall. Because the shear strength of masonry walls depends on the ratio M_u / (V_u d_v) and on the axial load P_u, different material definitions are required for the various wall segments involved. For this purpose, the values of M_u, V_u, and P_u are obtained from the linear-response model used as a basis to create the nonlinear model. The nonlinear stress-strain idealization used for shear is meant to represent the combined effects of masonry and shear reinforcement. This modeling approach is not intended to simulate realistic shear behavior but to help identify the wall segments that reach their shear strength (based on the MSJC [1] code) before their flexural strength.

The thickness of the layer representing masonry in compression or shear is the actual wall thickness. The thickness of the layer representing the flexural and axial reinforcing steel is defined by the steel area divided by the length of area element represented. The definition of a layer also requires assigning a material angle. For instance, an area element with nonlinear layers in Fig. (**3**) representing the reinforced masonry of the wall configuration in Fig. (**1**), should incorporate a layer of masonry material with nonlinear capabilities in the local 2-2 direction (or vertical direction) while linear-response is assigned to the local 1-1 direction (or horizontal direction). The material representing the flexural and axial reinforcement incorporates a layer of steel with nonlinear capabilities in the local 2-2 direction. The nonlinear material to represent masonry in shear is assigned only to the local 1-2 direction. Because the nonlinear layers are defined to represent membrane behavior, it is sufficient to assign a single integration (sampling) point in the thickness direction of each layer. For more details, see CSI [2].

To proceed with nonlinear static analyses for lateral loads, a gravity load case needs to be defined as a pre-load condition to determine the starting points on the stress-strain curves of each nonlinear layer.

3. NONLINEAR LINK MODEL

The nonlinear link element is a special type of line element that allows the modeling of material nonlinearity by means of user-defined force-deformation relationships. The area elements representing the interface of wall segments, where yielding is likely to occur, are replaced with nonlinear links, see Fig. (**7**). The area elements outside the assumed yielding regions are modeled with linear-elastic area elements using full gross section properties.

The force-deformation relationships assigned to the nonlinear links, to represent both axial and in-plane shear behavior of the yielding wall segments, are defined as Multilinear Plastic. The longitudinal direction of the link defines the axial behavior while the transverse direction defines the shear behavior. The force vs. deformation data depend on the tributary area of wall represented by each of the nonlinear links. The axial response characteristics of the nonlinear links directly control the flexural behavior of the wall. A detailed description of the advanced features of the Link element is presented by CSI [2].

For the nonlinear link to simulate axial response in compres-

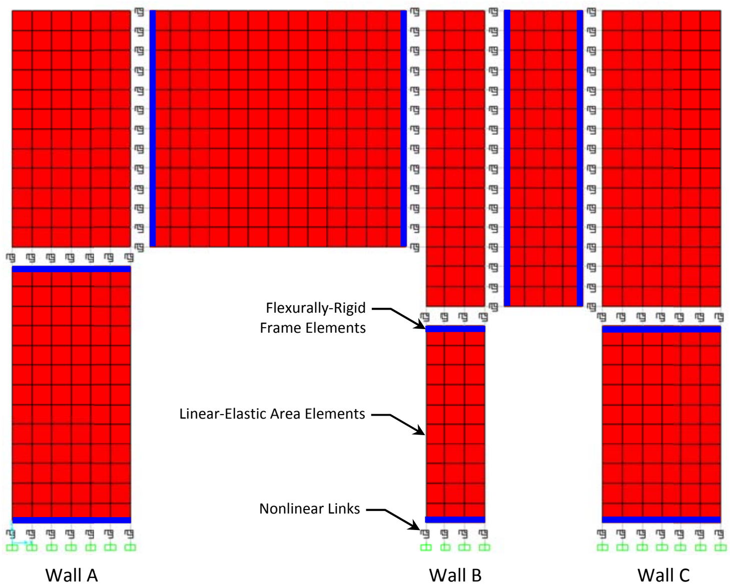

Fig. (7). Nonlinear Link Model.

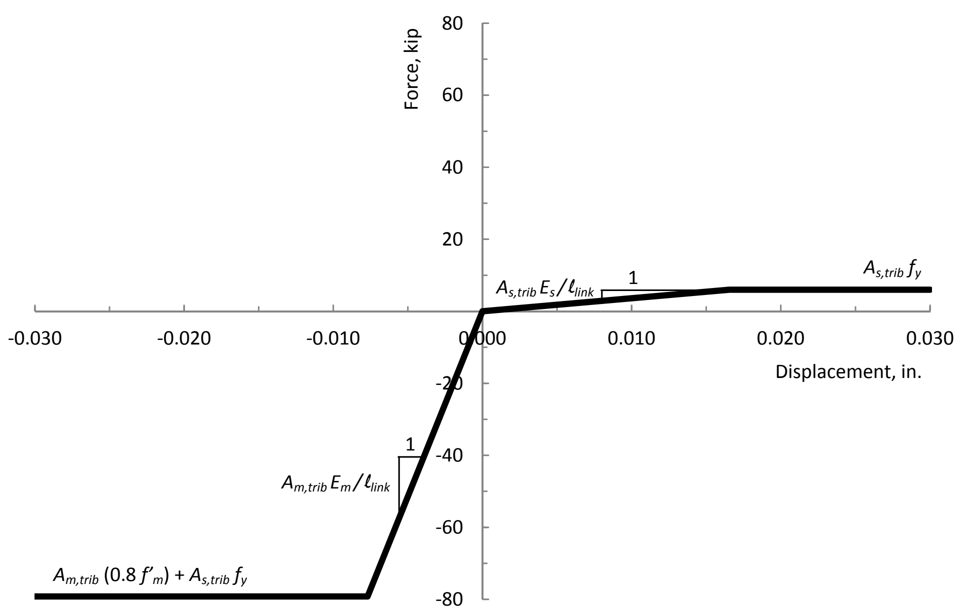

Fig. (8). Nonlinear Link Definition, Axial Direction, First Interior Links in Walls A and C (1 in. = 25.4 mm, 1 kip = 4.45 kN).

sion, the force-deformation curves need to account for the contributions of both masonry and reinforcement. To simulate axial response in tension, the contribution of masonry is neglected. Typical force-deformation curves are represented as bilinear on both the tension and compression quadrants, see Fig. (**8**). The initial stiffness in compression is based on the rigidity of masonry and the length of the nonlinear link. Analogously, the initial stiffness in tension is based on the rigidity of the steel reinforcement and the length of the nonlinear link. Peak forces are obtained after assigning 0.8 f'_m to the masonry in compression and f_y to the reinforcing steel in tension and compression. The post-yield stiffness, in tension and compression, is taken as zero. For links representing unreinforced masonry, the tension quadrant is defined using a nearly horizontal line with an effectively zero force. The nearly zero slope of the force-deformation curves at large deformations ensures a stable structure throughout the nonlinear analysis. The user must verify the output is limited to realistic displacements.

To simulate the response in shear, the force-deformation relationships assigned to the nonlinear links are defined as bilinear and symmetrical. The first line is defined by the stiffness based on gross section properties and the second line is horizontal (constant force) representing the nominal shear strength of the wall segment, see Fig. (**9**). The links representing nonlinear shear response are defined so that the shear carried by the link generates a secondary moment only at one end of the link. To properly account for the effects of this moment, a flexurally-rigid line element is added to fully engage the wall cross section, see Fig. (**7**). Careful attention is given to the orientation of the link local axes to deal with the secondary moment.

To proceed with nonlinear static analyses for lateral loads, a gravity load case needs to be defined as a pre-load condition to determine the starting points on the force-deformation curves of each nonlinear link.

4. NONLINEAR ANALYSIS RESULTS

The structure is analyzed nonlinearly for two lateral load cases, eastward and westward loading. Global shear (base shear) and local shear (per vertical wall segment) are monitored against the roof displacement, see Figs. (**10** to **13**). Each nonlinear static analysis has three main objectives: (1) identify where yielding occurs; (2) identify the type of nonlinear action (flexure or shear) that limits the force contribution of the yielding elements; and (3) determine the plastic base-shear strength. Nonlinear static analyses using the proposed simplified models are not intended for directly determining deformation demands or deformation capacities.

The simplified models considered only two types of nonlinear actions: flexure and shear. To identify the wall segments responding nonlinearly, the user needs to monitor the forces in the regions where nonlinear elements were assigned and check if the limiting strength of the nonlinear layers or links was reached.

The plastic base-shear strength, V_p, of the wall configuration may be determined using the base shear vs. roof displacement curves that result from the nonlinear static analyses (Figs. **10** to **13**). On each figure, an open circle is used to identify the last point on the curve where the slope exceeds 5% of the slope associated with the initial stiffness. The initial stiffness was obtained from linear-elastic response using gross section properties. The plastic base-shear strength so defined corresponds to the instance at which the structure has nearly developed a plastic mechanism. However, the limiting base-shear strength, V_{lim}, may be lower than V_p after consideration of the deformation capacities of the yielding wall segments which depend on whether the wall segments are controlled by flexure or shear. Deformation capacities are determined using the simple rules in Table **1**, Section X.3.

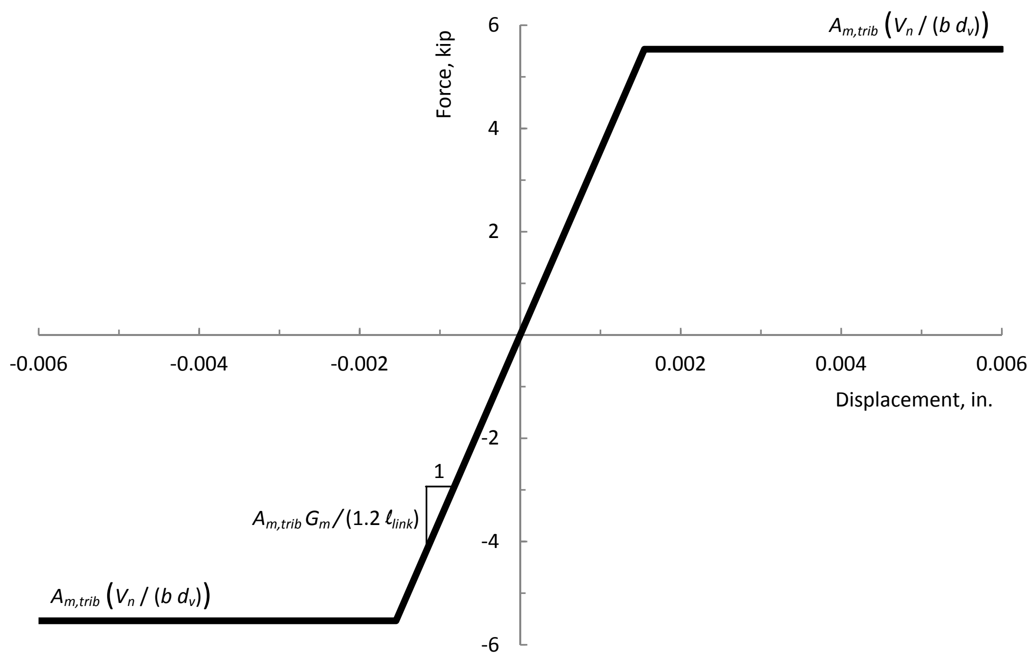

Fig. (9). Nonlinear Link Definition, Shear Direction, Interior Links in Walls A and C (1 in. = 25.4 mm, 1 kip = 4.45 kN).

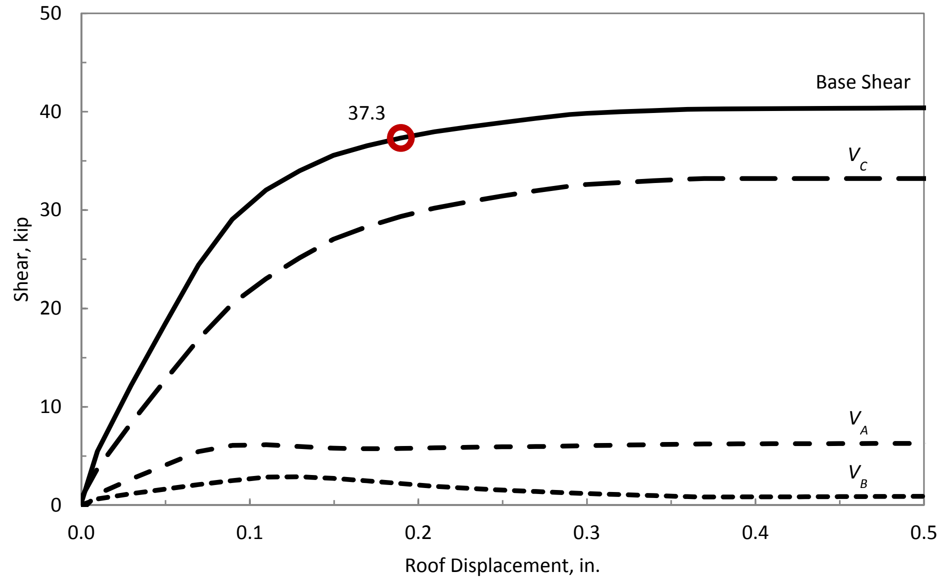

Fig. (10). Shear vs. Roof Displacement for Nonlinear LAYER MODEL, EASTWARD Loading (1 in. = 25.4 mm, 1 kip = 4.45 kN).

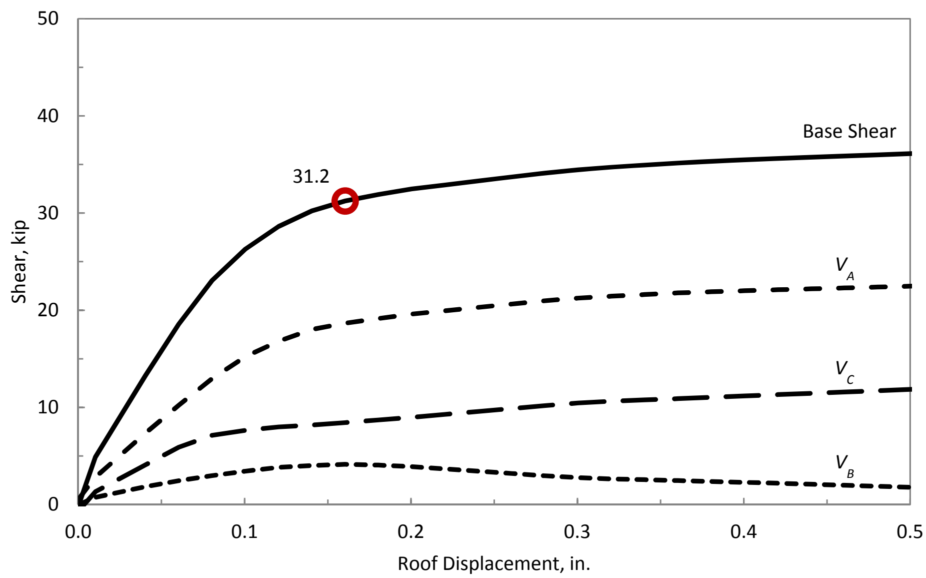

Fig. (11). Shear vs. Roof Displacement for Nonlinear LAYER MODEL, WESTWARD Loading (1 in. = 25.4 mm, 1 kip = 4.45 kN).

For a shear-controlled wall segment, where the shear demand exceeds half of its nominal shear strength, the deformation capacity is limited to h_w / 200 or h_w / 400 depending on the amount and detailing of reinforcement, refer to Section X.3.2 in the Limit Design code, see Table **1**. The use of half the nominal shear strength is mainly to account for flexural overstrength, refer to Section X.1(d) in Table **1**.

For the wall segments responding nonlinearly, the proposed computer models directly account for the interaction between axial forces (*P*) and moments (*M*). In addition to the effects of gravity loads, the axial forces vary due to an increase in lateral loads. The *P*-*M* interaction causes an increase in moment capacity in the wall segments resisting additional compression induced by lateral loads.

The maximum moments that develop in the wall segments may also be limited by the shear capacity assigned to layers and links. For this purpose, it was decided to assign the shear capacity that corresponds to the full nominal shear strength (without the ½ multiplier of Section X.1(d) in Table **1**). The full-strength assignment allows the nonlinear analysis to identify the wall segments that develop their nominal shear strength. For simplicity, the assigned shear capacity was based on the axial forces indicated in Table **1** Section X.1(c).

For a flexure-controlled wall segment, where the shear demand is below half of its nominal strength, the deformation capacity is derived based on a plastic rotation capacity of $\phi_u\ l_p$ = (ε_{mu} / c) (0.5 l_w), where c is calculated for the axial load due to load combination 5 per Section 2.3.2 of ASCE 7 (i.e., 1.2D+1.0E+L+0.25S)[4]. The use of $\phi_u\ l_p$ is consistent with the Strength Design provisions of MSJC [1] to check the need for special boundary elements in special reinforced masonry shear walls. Similar provisions apply to special reinforced concrete shear walls in ACI 318 [5]. Note that this deformation capacity only includes the contributions

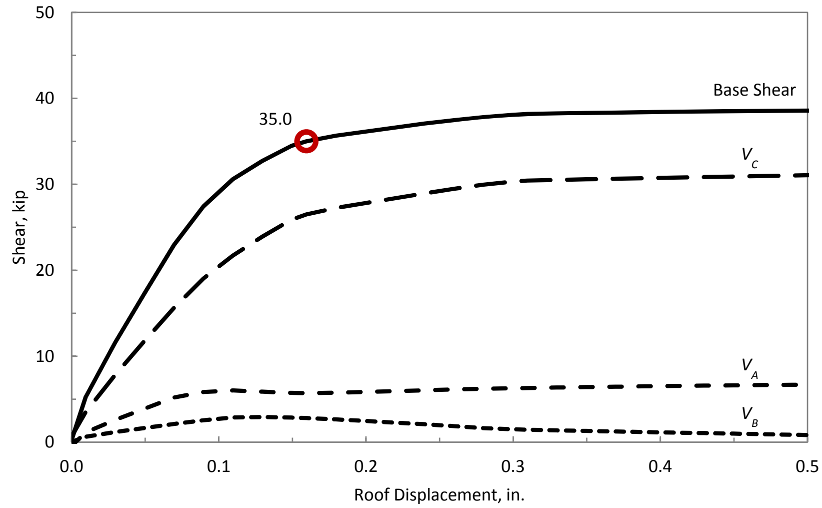

Fig. (12). Shear vs. Roof Displacement for Nonlinear Link Model, Eastward Loading (1 in. = 25.4 mm, 1 kip = 4.45 kN).

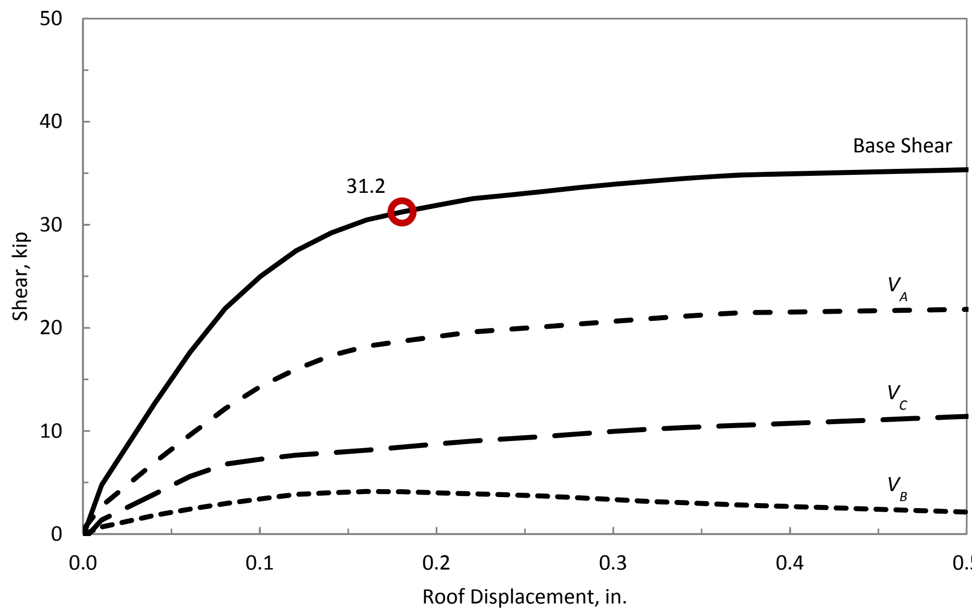

Fig. (13). Shear vs. Roof Displacement for Nonlinear Link Model, Westward Loading (1 in. = 25.4 mm, 1 kip = 4.45 kN).

of curvature over the plastic hinge regions and is to be checked against total (elastic plus plastic) deformation demand. Neglecting the contribution of elastic curvature outside the plastic hinge regions leads to lower deformation capacity, a conservative approach. Neglecting deformations due to shear and bond slip also adds conservatism.

The limiting base-shear strength, V_{lim}, is defined after consideration of the deformation capacity of each of the wall segments responding nonlinearly. For the wall configuration to be considered code compliant, V_{lim}, times the strength reduction factor (ϕ = 0.8) shall exceed the design shear at the base of the wall, V_{ub}, calculated after consideration of the seismic design requirements for building structures in ASCE 7 [4], see Table **1** Section X.2.

For the wall configuration in Fig. (**1**), the controlling plastic base-shear strength, V_p, corresponds to westward loading, see Figs. (**10** to **13**). Both the Nonlinear Layer model and the Nonlinear Link model indicate V_p = 31 kip (138 kN). The controlling deformation capacity corresponds to the shear-controlled condition of Wall C under eastward loading, for which the design roof displacement, δ_u, needs to be controlled to $\delta_{cap} = h_w / 400$ = 0.24 in. (6.1 mm). The limiting base-shear strength is to be determined using $V_{lim} = V_p (\delta_{cap} / \delta_u)$, with $\delta_{cap} / \delta_u \leq 1$. The term δ_{cap} / δ_u is justified because the design displacement, δ_u, is calculated using linear-elastic analysis and therefore a reduction of displacement implies a proportional reduction of force.

The Limit Design code (Table **1**) leads to the limiting mechanism shown in Fig. (**14**) with a base-shear strength of 31 kip (138 kN). The calculations shown in Fig. (**14**) are based on hinge strengths corresponding to the axial forces due to gravity loads and therefore only one direction of roof displacement needs to be considered. It is important to note

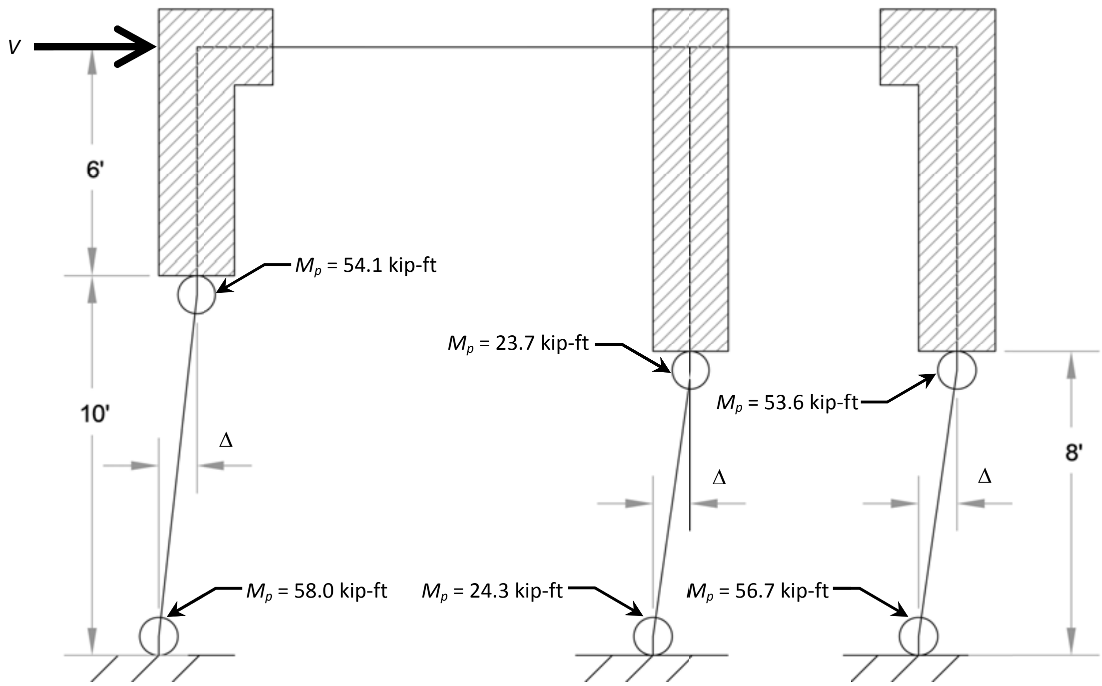

Fig. (14). Controlling Yield Mechanism (1 ft = 305 mm, 1 kip = 4.45 kN).

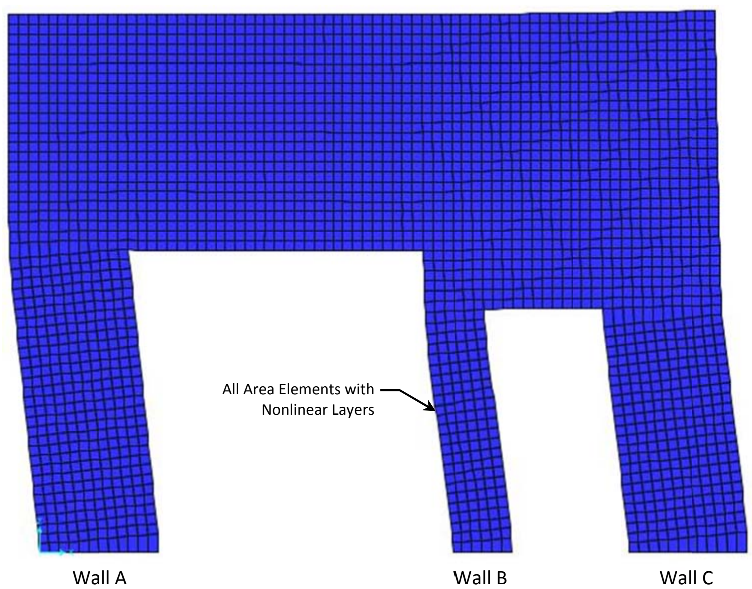

Fig. (15). Deformed Shape for Refined Nonlinear Layer Model, Westward Loading

that for this example, ignoring the effects of axial loads due to lateral loads when determining the hinge strengths led to the same base-shear strength as in the computer models where axial load effects due to lateral loads are accounted for. This safe outcome for the proposed Limit Design code implies that for the wall configuration considered, the increase in flexural strength of the compression wall segments was offset by a reduction in flexural strength of the tension wall segments.

To evaluate the merits of the proposed simplified Nonlinear Layer model, a refined model was developed. The refined model incorporates the tensile strength of masonry and assumes zero post-peak residual stress. The refined model also extended the nonlinear area elements throughout the clear length and joints of the wall segments. All area elements included nonlinear layer definitions. In addition, the mesh size was reduced to 4 in. by 4 in. (102 mm by 102 mm). The modeling of the joint used new material defini-

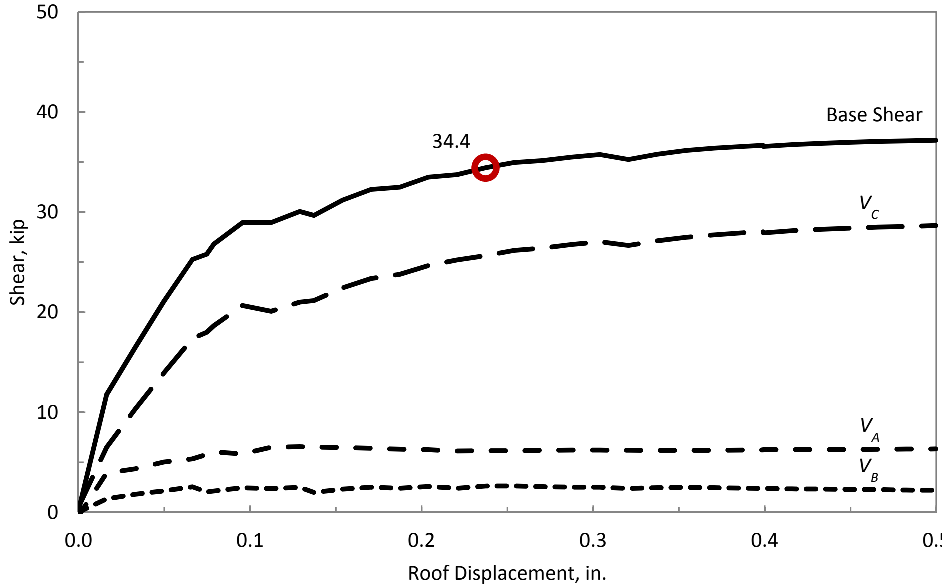

Fig. (16). Shear vs. Roof Displacement for Refined Nonlinear Layer Model, Eastward Loading (1 in. = 25.4 mm, 1 kip = 4.45 kN).

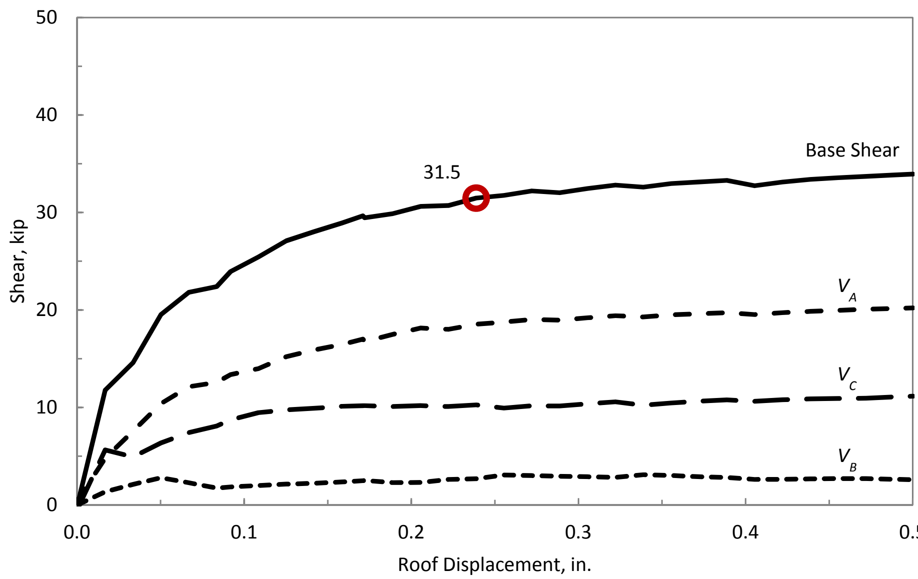

Fig. (17). Shear vs. Roof Displacement for Refined Nonlinear Layer Model, Westward Loading (1 in. = 25.4 mm, 1 kip = 4.45 kN).

tions to account for the higher shear strength of the joint and to consider the effects of the intersecting longitudinal reinforcement. Results of the nonlinear static analyses using the refined model are shown in Figs. (**15** to **17**). Although the run time increased by a factor of about 10, the resulting base-shear strength nearly coincided with the values obtained using the simplified models.

5. CONCLUSION

The two simplified models, the Nonlinear Layer model and the Nonlinear Link model, for calculating the nonlinear static response of reinforced masonry shear walls gave satisfactory results when compared to the analysis output from a refined analytical model applied to a perforated wall configuration. Additional case studies are under development to further test these models including cases of walls with flanges.

The computer models presented were developed to help evaluate the merits of a new Limit Design method under consideration for future versions of the MSJC Masonry Building code [1]. The modeling approach may also be useful in the evaluation of existing buildings for seismic rehabilitation.

APPENDIX A. NOTATION

The following symbols are used in this paper:

A_g	=	gross cross-sectional area of member
$A_{m,trib}$	=	tributary area of masonry to nonlinear link
$A_{s,trib}$	=	tributary area of reinforcement to nonlinear link
b	=	width of section
c	=	distance from extreme compression fiber to neutral axis

C_d = deflection amplification factor specified in ASCE 7 [4]

d_v = actual depth of member in direction of shear considered

E_m = modulus of elasticity of masonry

f'_m = specified compressive strength of masonry

f_y = specified yield strength of reinforcement

G_m = shear modulus of masonry

h_w = clear height of vertical wall segment or clear span of horizontal wall segment

l_{link} = length of nonlinear link

l_p = plastic hinge length, taken as 0.5 l_w, not to exceed 0.5 h_w

l_w = length of wall segment in direction of shear force

M_n = nominal flexural moment strength

M_p = flexural moment strength assigned to plastic hinge

M_u = factored flexural moment

P_u = factored axial force

V_{lim} = limiting base-shear strength of a line of lateral load resistance after consideration of the deformation capacities of the yielding wall segments

V_n = nominal shear strength

V_p = plastic base-shear strength of a line of lateral load resistance

V_u = factored shear force

V_{ub} = factored base-shear demand on a line of lateral load resistance calculated in accordance with ASCE 7 [4]

δ_u = design displacement

Δ = virtual roof displacement

ε_{mu} = maximum usable compressive strain of masonry, 0.0035 for clay masonry and 0.0025 for concrete masonry

ϕ = strength reduction factor

ϕ_u = ultimate curvature, equal to ε_{mu} divided by c

CONFLICTS OF INTEREST

The authors confirm that this article content has no conflicts of interest.

ACKNOWLEDGMENTS

The work described in this manuscript was sponsored by The NCMA Education and Research Foundation. The writers are indebted to Steve Dill (KPFF Consulting Engineers, Seattle, WA) and Jeff Dragovich (NIST, Gaithersburg, MD) for their valuable input during this study.

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Received: September 21, 2011 Revised: August 04, 2012 Accepted: September 05, 2012