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The Proof of Gravity Model with Negative Exponential Land-mixed Entropy and Similar to the Method of Hyman Calibration Technology

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Abstract: Mixed Land use is a common concept used in urban planning and urban transportation. When changing the layout of the land in urban area, essential maintenance of trip demand between local and nonlocal balance in transportation planning becomes a problem. On the basis of the degree of land mixing shown with entropy function, this article makes use of maximum entropy principle which deduces the double restraint gravity model with the parameters of mixed land-use entropy. It also discusses the calibration of the model and problems in practical application. The research work carried out in this article indicates that trip distribution of gravity model described belongs to a special kind of simple giant system, while the mathematical structure of parameters of land-mixed entropy model carries a negative exponential. This result has certain theory value in promotion of the coordinated planning analysis of land use and transportation.

Keywords: Gravity model, land-mixed entropy, maximum entropy principle, the coordinated planning of land use and transportation.

1. INTRODUCTION

Urban transportation demand is directly affected by the degree of land use mixing. If the parameters are modeled by the maximum entropy principle which describes the degree of land use mixing, and are directly introduced to the gravity model, this may play an important role in optimizing the land use and card traffic demand, and can also be directly used for "hybrid land development" planning scheme and judgment. However, at present, the study of such problems is rare.

Maximum entropy principle established the theoretical foundation of trip distribution gravity model, and is one of the important methods according to the practical problems derived from the simulation time and space distribution of traffic demand. Regarding the issue, a lot of research results are present at home and abroad. Different travel distance distribution studies of urban land use have formed the characteristics using the method of computer simulation by gravity model, and by putting forward the second order Erlang distribution for simulating the trip distance distribution of the city [1, 2]. Yao Ronghuan and Wang Dianhai proposed a probability torque estimation model by using principle of maximum entropy under the constraint conditions of mathematical statistics of the average moments, which can be used to estimate trip distribution [3, 4]. Theodore Tsekeris and Antony Stathopoulos further proposed dynamic gravity model and conducted an analysis on the difference of the single and double dynamic gravity model [5, 6]. Deng Mingjun and Wang Tie-zhong presented an improved gravity model with actuality OD, the nature of the land, and trip cost linear weightage, also with a higher estimation precision than general gravity model [7]. Moreover, Xiang Li *et al.* proposed the trip distribution models with fuzzy cost constraints and random trip generation using maximum entropy principle and presented hybrid intelligent algorithm [8]. Ma Jing *et al.* brought land-mixed entropy into the gravity model, proposing a generalized form of uniform gravity model with inside and outside trip distributions [9]. The above documents carry important reference value for the research conducted in this paper.

This article proves the double restraint gravitational model with negative exponential land-mixed entropy by using the maximum entropy principle through the carding of the concept of entropy, and gives the corresponding parameter calibration algorithm and the application example.

2. THE MOST PROBABLE DISTRIBUTION AND THE MAXIMUM ENTROPY PRINCIPLE

2.1. The Most Probable Distribution

It is universally known that a coin possesses two sides, and if a coin is continuously tossed, the probability for both positive and negative becomes 0.5. Therefore, if 4 coins are thrown at a time, following 16 kinds of results can be obtained (Fig. (**1**)).

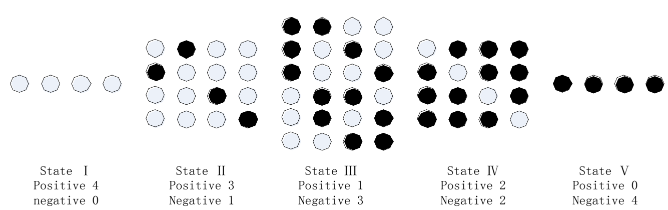

Fig. (1). Four coins coin experiment.

In the results mentioned above, if the order between the coin is not considered, and five different states are obtained:

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①All positives and ②All negatives with probability 1/16;③ 1 positive and 3 negatives or④3 positives and 1 negative: 4/16;⑤2 positives and 2 negatives with probability 6/16. It has been noticed that if each coin without interfering with each other, and both the probabilities of positive and negative are equal, Every time the four pieces of flip a coin for the probability of a certain state associated with the state of the number of how many. So in fact, a microscopic (individual) distribution and a relationship between the macroscopic risks can be established. In this case, if there are 4 COINS at a time, the most likely state with the largest probability will be of 2 positives and 2 negatives. If the number of micro individuals constituting the system gradually increases, this "possible" gradually becomes the "inevitable".

2.2. The Maximum Entropy Principle

To illustrate this problem, the probability of the state largely known as "the most probable state", with the rest of the state being "the general state", introduces a parameter ε, used to represent the ratio of the most probable state probability and general state average probability. Let the number of COINS in the example above be as illustrated in Table **1**.

Table 1. N-Coins experiment's relationship with the most probable state.

Number of coins	The total number of states	Probability of the most probable state	ε
n=2	4	0.500	2
n=4	16	0.375	6
n=20	1.048×10^{6}	0.176	1.848×10^{5}
n=40	1.100×10^{12}	0.125	1.378×10^{11}
n=200	1.607×10^{60}	0.056	9.055×10^{58}
n=400	2.582×10^{120}	0.040	1.030×10^{119}

It can be seen from the above example that when there is a gradual increase in the quantity of the throwing COINS, the most probable state probability gradually occupies the absolute advantage. Otherwise in a coin-operated experiment, it is inevitable. This phenomenon can be qualitatively described as: for system consisting of a large number of random elements, the actual macroscopic state contains the most microscopic ones. This principle is often referred to as “the maximum entropy principle”. Considering that the gas since is composed of a large number of particles (molecules), it must also be in compliance with the principle of maximum entropy.

3. PROBABILITY MEASURE ENTROPY AND SIMPLE GIANT SYSTEM STATE DESCRIPTIONS

3.1. Probability Measure Entropy

Assuming that a sample space ***X*** has ***n*** events of X_i, the probability measure of each event is P_i**(i=1,2,...,n)**, and there is a basic relationship $\Sigma_i P_i=1,\ P_i\geq0$. If there is a function $S(P_1,P_2,...,P_i)$, the following conditions will be satisfied:

(1) For a fixed ***n***, ***Sn*** is continuous function of $P_1,P_2,...,P_i$;

(2) If P_i =1/n$P_i=\frac{1}{n}$, the corresponding S(1/n,1/n,...,1/n) $S\left(\frac{1}{n},\frac{1}{n},\cdots,\frac{1}{n}\right)$is the monotone increasing function of ***n***;

(3) If a test is decomposed into several successive tests, then the S weighted sum is equal to the original value S;

This moment, function $S(P_1,P_2,...,P_i)$ is decided as entropy, and has a unique expression:

$$S=-k\sum_i P_i\log_x p_i \tag{1}$$

After understanding the definition of general concept, it can be discussed in the system of science.

3.2. Simple Giant System State Descriptions

In order to more intuitively show this problem, the starting must be from the simplest case. Suppose there are two elements of system, the probability of each appear respectively as P_1 and P_2, having$P_1+P_2=1$. Let P_1 change from 0 to 1 with fixed step, and note that at this time $P_2=1-P_1$. So there are:

$$S=P_1\ln P_1+P_2\ln P_2 \tag{2}$$

$$S=P_1\ln P_1+(1-P_1)\ln(1-P_1) \tag{3}$$

When $P_1=0$, this system is constituted by P_2, the system entropy is zero, and there is no uncertainty. So as $P_1=1$.

When $P_1=P_2=0.5$, the possibility of two states P_1 and P_2 are same as 0.5. By this time, maximum entropy is largest also being the biggest uncertainty. Its function image is shown in Fig. (**2**).

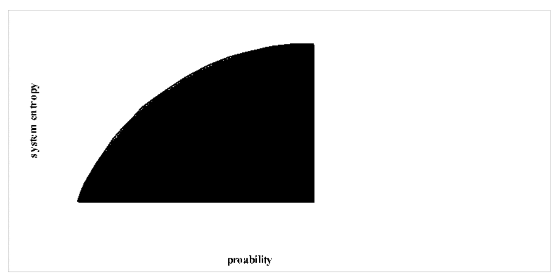

Fig. (2). The images of two variables entropy function.

From the above example, it can be seen that the entropy function is very special. If the order of above events is not considered as 1 and 2, the entropy function can in fact use a single numerical unique difference between system elements constituting a state. In other words, there is one-to-one correspondence between the system entropy and its distribution structure of the elements. The entropy function can be briefly used to describe a very complex simple giant system.

According to the definition of system science, "simple" of "simple giant system" refers to interaction being simple; "Giant" refers to a huge number of the elements of a system so each element cannot be analyzed, otherwise it makes little sense to the analysis of control system of macroscopic state. In life, there are many systems which can be classified as simple giant systems, such as trip distribution system.

3.3. Trip Distribution as Simple Giant System

Trip distribution in traffic planning is an important part of traffic demand analysis. The problem which requires the main solution is where "comes from" and where "goes", which is in a direct relationship in the urban spatial structure, land layout state and road traffic network (Fig. **3**).

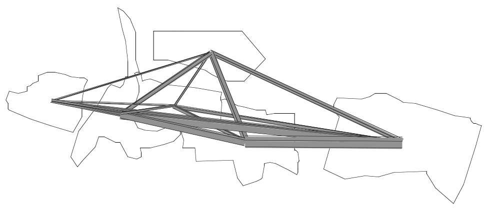

Fig. (3). Planning year travel desire line diagram in a city.

The original gravity model was proposed by planner Casey (Casey, 1955), to analyze an area shopping trip activity between different towns, having a basic form as follows:

$$t_{ij} = \frac{kG_iA_j}{d_{ij}^2} \quad (4)$$

Where, $t_{ij}t_{ij}$ is trip volume from $ii$to j; G_ij; G_i is trip generation volume; A_j is trip attract volume; d_{ij} is the distance between i and j; k is the parameter.

Careful comparison reveals that the model and the physical application of Newton's universal gravitation formula, actually is the imitation of the formula for gravity. Gravity model later after a series of developments has become one of the categories of the floorboard of the model. In urban transportation planning at present, the most common trip distribution model is the double restraint gravitational model of impedance function of negative exponent, having the basic forms as:

$$t_{ij} = k_i k_j^{'} G_i A_j e^{-\theta c_{ij}} \quad (5)$$

$$k_i = \left[\sum_j k_j^{'} A_j e^{-\theta c_{ij}}\right]^{-1} \quad (6)$$

$$k_j^{'} = \left[\sum_i k_i^{'} G_i e^{-\theta c_{ij}}\right]^{-1} \quad (7)$$

where, G_i is trip generation volume; A_jA_j is trip attract volume;$e^{-\theta c_{ij}}$ $e^{-\theta c_{ij}}$ is the impedance of the negative exponential function, c_{ij} c_{ij} is the impedance, usually showed by trip distance or time cost, θ is the parameter of impedance function on behalf of the end sensitivity of the choice of travel time. k_i, k_j are the iterative parameters respectively and the intermediate variables.

In fact, the gravity model expresses only two things for practical significance:

①in the same situation, the nearer the starting point is selected, the farther is the destination;

②the opportunity of being large trip volume is larger between the points of departure generation volume and destination abstract volume, both of which are large.

According to the previous discussion, it can be seen that the trip distribution defined by the gravity model accords with the definition of simple giant system.

4. THE PROOF OF LAND-MİXED ENTROPY AND GRAVİTY MODEL OF ENTROPY

4.1. Definition of Land-Mixed Entropy and Union of Land-Mixed Entropy

4.1.1. Land-Mixed Entropy

Land-mixed entropy: using entropy model, measures the degree of various types of land use mixed within a certain area (Table **2**). A city partition of land-mixed entropy is:

$$H_i = -K_1 \sum_{k=1}^{n_i} \left(p_{i,k} \times \ln p_{i,k}\right) \quad (8)$$

Where, H_i is the city partition i of land-mixed entropy, characterization of travel (attract) from the possibility of size falls in this area; k_1 is parameter related to the units of measurement used in the variable; $p_{i,k}$ $p_{i,k}$ is ratio of land use type k in partition i, %. Calculation is:

$$p_{i,k} = \frac{A_{i,k} \cdot \rho_{i,k}}{\sum_m A_{i,m} \cdot \rho_{i,m}} \quad (9)$$

Table 2. Planning year peak–time trip amount in a city (Unit: person/h).

A P	SHAN-G MA YING	PAN LONG	CHENG CANG	FU TAN	DAI NA	JIN WEI
SHANG MA YING	3241	9855	18490	5354	16331	7345
PAN LONG	3174	6313	16584	4433	14026	5910
CHEN CANG	6818	18987	6542	10166	33397	12573
FU TAN	1866	4797	9609	5432	8287	4233
DAI MA	8778	23412	48687	12781	6458	15771
JIN WEI	9124	22794	42357	15087	36445	4531

Note: A is the area of land. P is ratio of land use type.

$A_{i,m}$ $A_{i,m}$ is the area of land using type k in traffic zone i, km^2; $p_{i,m}\rho_{i,m}$ is the average plot ratio of land using type k in traffic zone i, by taking the average limit according to the present situation investigation or plan.

With this index, the distinguished combinations of different land use can be made. For example, two pieces of planning land (Figs. **4** and **5**). It is difficult to distinguish the mixed degree high and low from the intuitive sense of block 1 and block 2, But from land-mixed entropy, this can be obtained:H1=0.847226, H2=1.036542. Obviously, the degree of land mixed in block 2 is far larger than block 1.

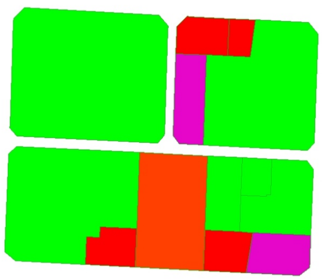

Fig. (4). Block 1 the layout of land use (H1=0.847226).

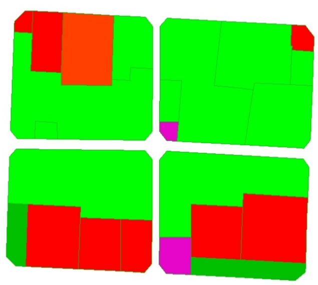

Fig. (5). Block 2 the layout of land use (H2=1.036542).

So far, there had been a tool which could distinguish the mixed degree between different land uses in different partitions. But there are problems in using the tool for analysis of traffic distribution. Since generation point and abstract point may be the same point in urban transportation division system, called as trips in the inner zone or may be different points known as Interval trip.

4.1.2. Union Of Land-Mixed Entropy

In order to describe the land mixed degree of start point and end point at the same time, an indicator "union of land-mixed entropy" shall be built. For a city traffic zone i and j, index of union of land-mixed entropy is:

$$H_{ij} = -K_1 \sum_{k=1}^{n_{ij}} \left(p_{ij,k} \times \ln p_{ij,k} \right) \quad (10)$$

Where, H_{ij} is the union of land-mixed entropy of traffic zone i and j, showing the size of the possibility of travel (attract) from fall in two areas; K_1 is parameter related to the units of measurement used in the variable; $p_{ij,k}$ is the ratio of land use type k in traffic zone i and j. Calculation is:

$$p_{ij,k} = \frac{A_{ij,k} \cdot \rho_{ij,k}}{\sum_m A_{ij,m} \cdot \rho_{ij,m}} \quad (11)$$

$A_{ij,m}$ is the area of land use type k in traffic zone i and j, km^2; $\rho_{ij,m}$ is the average plot ratio of land use type k in traffic zone i and j, the average limit is taken according to the present situation investigation or plan.

4.2. The Proof of Land-Mixed Entropy Gravity Model

Assuming that in a trip distribution totally having Q trips, every trip can choose OD random. So, there are n×n choices for one trip. If the trip volume is q_{ij} from i to j, it is not known how much q_{ij} are present, but it can formally provide the travel choice on above average, the possibility from i to j is $p_{ij} = q_{ij}/Q$.

Then, by considering the whole trip distribution entropy:

$$H(x) = -\sum_{ij}^{n} \left[\frac{q_{ij}}{Q} \ln\left(\frac{q_{ij}}{Q} \right) \right] \quad (12)$$

$$= -\frac{1}{Q} \sum_{ij}^{n} q_{ij} \left(\ln q_{ij} - \ln Q \right) \quad (13)$$

According to the maximum entropy principle discussed earlier, it can be known: By each of the trips randomly selected in Q, eventually on macro performance, surely one of the largest entropy is:

$$\max_x H(x) = -\frac{1}{Q} \sum_{ij}^{n} q_{ij} \left(\ln q_{ij} - \ln Q \right) \quad (14)$$

To express in an easier way, constant 1/Q is avoided.

The row and column of trips should be the same, there is:

$$\sum_j q_{ij} = P_i \quad (15)$$

$$\sum_i q_{ij} = A_j \quad (16)$$

Considering that the impedance of start and end points has a certain influence on the probability of selection point actually. To assume the impedance is c_{ij} between i and j, so the total trip time cost should be conserved:

$$\sum_i q_{ij} \times c_{ij} = Q \times \bar{c} = C \quad (17)$$

At the same time, considering that the characteristics of the entropy function are drab and additive, so the overall land use entropy should be conserved. Assuming that the union of land-mixed entropy of traffic zone i and j is h_{ij}, it similarly has:

$$\sum_i q_{ij} \times h_{ij} = Q \times \bar{h} = H \quad (18)$$

Formula (14) as target function, formulae (15)-(17) as constraints, q_{ij}、c_{ij}、h_{ij} being non-negative having must practical significance, and all these establishing mathematical programming as follows:

$$\max_x H(x) = -\sum_{ij}^{n} q_{ij} \left(\ln q_{ij} - \ln Q \right) \quad (19)$$

$$s.t.\begin{cases}\sum_j q_{ij} = P_i & (20)\\ \sum_i q_{ij} = A_i & (21)\\ \sum_{ij} q_{ij}\times c_{ij} = C & (22)\\ \sum_{ij} q_{ij}\times h_{ij} = H & (23)\\ q_{ij}\ge 0, c_{ij}\ge 0, h_{ij}\ge 0 & (24)\end{cases}$$

The result of this plan is the trip distribution which is required.

As formulae (20) and (21) have n individuals (n is the number of rows or columns for trip matrix), in order to solute this plan, the Lagrange function can be constructed as follows:

$$\begin{aligned}\mathcal{L} = &-\sum_{ij}^{n} q_{ij}\left(\ln q_{ij} - \ln Q\right) + \sum_i \alpha_i\left(P_i - \sum_j q_{ij}\right)\\ &+\sum_j \beta_j\left(A_j - \sum_i q_{ij}\right) + \theta\left(C - \sum_{ij} q_{ij}\times c_{ij}\right)\\ &+\gamma\left(H - \sum_{ij} q_{ij}\times h_{ij}\right)\end{aligned} \quad (25)$$

$\alpha_i, \beta_j, \theta, \gamma$ are Lagrange coefficient.

Let L derivative with q_{ij}, and make it to 0, there is:

$$\begin{aligned}\mathcal{L}' = &-\ln q_{ij} + \ln Q - 1 - \alpha_i\\ &-\beta_j - \theta c_{ij} - \gamma h_{ij} = 0\end{aligned} \quad (26)$$

Solution is:

$$q_{ij} = \frac{Q}{e} e^{-\alpha_i} e^{-\beta_j} e^{-\theta c_{ij}} e^{-\gamma h_{ij}} \quad (27)$$

Change this formula. Because:

$$k_i = \frac{Q}{P_i e}\mathrm{e}^{-\alpha_i}, k_j^{'} = \frac{e^{-\beta_j}}{A_j} \quad (28)$$

And get:

$$q_{ij} = k_i k_j^{'} P_i A_j e^{-\theta c_{ij}} e^{-\gamma h_{ij}} \quad (29)$$

In order to distinguish from the general double restraint gravitational model, formula (29) land-mixed entropy gravity model can be used.

4.3. Algorithm and Example

In order to compare the difference between the land-mixed entropy gravity model and general double restraint gravitational model, formula (29) needs to be rewritten:

$$q_{ij} = k_i k_j^{'} P_i A_j e^{-\left(\theta c_{ij} + \gamma h_{ij}\right)} \quad (30)$$

In comparison with the former formula (5) shown in general double restraint gravitational model, it can be seen that relative to the general gravity model, the land-mixed entropy gravity model adds the description of union of land-mixed entropy, as h_{ij}. It provides direct tool to describe influence of the degree of land-mixed on trip distribution.

4.3.1. Parameter Calibration

Step1: Since the number of iteration m=0, initial value given is:

$$\theta_0 = \frac{1}{\bar{c}}$$

$$\gamma_0 = \frac{1}{\bar{h}}$$

Step2: Since m=m+1, land-mixed entropy gravity model is used for distribution, for solving its average impedance $\bar{c}_m$ and average land-mixed entropy $\bar{h}_m$.

Step3: Revision of estimates.

If m=1, then:

$$\theta_1 = \frac{\bar{c}_1}{\bar{c}}\theta_0$$

$$\gamma_1 = \frac{\bar{h}_1}{\bar{h}}\gamma_0$$

If m>1, then:

$$\theta_m = \frac{\left(\bar{c} - \bar{c}_{m-1}\right)\theta_{m-1} - \left(\bar{c} - \bar{c}_m\right)\theta_m}{\bar{c}_m - \bar{c}_{m-1}}$$

$$\gamma_m = \frac{\left(\bar{h} - \bar{h}_{m-1}\right)\gamma_{m-1} - \left(\bar{h} - \bar{h}_m\right)\gamma_m}{\bar{h}_m - \bar{h}_{m-1}}$$

Step4: Given the margin of errorε_1, ε_2. If

$$\frac{\left|\bar{c} - \bar{c}_m\right|}{\bar{c}} \le \varepsilon_1$$

Then

$$\frac{\left|\bar{h} - h_m\right|}{\bar{h}} \le \varepsilon_2$$

is the terminating, and output θ_m, γ_m.

Else the process is switched on to Step2.

4.3.2. Calculation Example

Assuming that there are three traffic zones of distribution system, including the traffic zones in between as shown in Table **3**, the trip impedance and the in-between traffic zones are shown in Tables **4** and **5**.

Table 3. Observation trip matrix (unit: m/h).

P \ A	1	2	3
1	50	10	25
2	11	35	14
3	24	15	20

Note: A is the area of land. P is ratio of land use type.

Table 4. Observation trip impedance (unit: h/m).

A / P	1	2	3
1	1.5	3.0	2.5
2	3.0	1.7	3.5
3	2.5	3.5	2.0

Note: A is the area of land. P is ratio of land use type.

Table 5. Observation union of land-mixed entropy.

A / P	1	2	3
1	2.07	1.88	1.66
2	1.88	1.69	1.47
3	1.66	1.47	1.25

Note: A is the area of land. P is ratio of land use type.

According to the algorithm to calculate θ=0.36, γ=0.34. At this time, estimate trip distribution is:

Table 6. Trip distribution simulation.

A / P	1	2	3
1	42	20	23
2	20	26	13
3	23	13	22

Note: A is the area of land. P is ratio of land use type.

At this time the average estimated error is 28.92% (Table **6**).

CONCLUSION

This paper discussed the algorithm and idea of the concept of entropy used in the description of degree of the land-mixed, by putting forward the idea of using land-mixed entropy in trip distribution, and through the maximum entropy principle that deduces the double restraint gravitational model. Then, it discussed the model parameters calibration algorithm. Through the research conducted in this article, the following results are obtained:

(1) Although the selection process of trip distribution start and end points are influenced by many factors, but as the trip distribution of the gravity model is described, it is clearly a special kind of simple giant system.

(2) Land-mixed entropy index presents negative exponential function structure in the gravity model.

(3) There is still a large estimation error in land-mixed entropy gravity model. But even so, due to the direct introduction of index of land use, it still holds certain theoretical value to promote the overall analysis of land use and transportation planning.

Overall, although this article discussed the mechanism of land-mixed entropy gravity model and the parameter calibration from two aspects of theory and practice, but based on the experimental results, the estimation precision of the model is still inadequate. Further research work should be carried out focusing on the improvement of the precision of the model.

CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

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