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Construction Monitoring of Cable-stayed Bridges Based on Gray Prediction Model

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Abstract: The construction process of cable-stayed bridges is complex and has many influencing parameters. Construction monitoring plays an important role in the construction process to ensure the structural safety and meet the design requirements. Gray system theory is applied to analyze and predict structural deformation and cable tension for construction monitoring, which regards the cable-stayed bridge under construction as an interferential gray system with a physical prototype and analyzes the random process as a gray process. The gray prediction model has been established by using and evaluating girder and cable tension as two control inputs of the system in the construction process of cable-stayed bridges. The girder and cable tension of subsequent construction stages were predicted, adjusted, and evaluated by using feedback information obtained from measuring and rectifying the gray prediction model to effectively control and adjust the bridge configuration and cable tension. Results show that gray prediction model has good prediction precision, which can control the structural configuration in the ideal state and meet the design requirements.

Keywords: Cable-stayed bridge, structure analysis, gray prediction model, construction control.

INTRODUCTION

A cable-stayed bridge is composed of inclined cables, tower, and girder and has the characteristics of large crossover ability, lightweight structure, large clearance height, and good earthquake resistance. Therefore, the cable-stayed bridge has been extensively adopted as a beautiful style of modern bridge structures. The girder configuration and internal structural force in the completed stage have a close relationship with the construction method because the cable-stayed bridge is a super high-order static structure. The construction process is not only complex (*e.g.,* tensioning the cables, moving the hanging basket, and pouring the concrete) but has many influencing parameters, such as the material elastic modulus, weight of the structure, and construction loads. Generally, these parameters are assumed ideal based on the design code at the design stage. In the construction process, errors between the actual and theoretical values are difficult to avoid because of various factors. If the errors are not adjusted and controlled in time, they accumulate and change the theoretical configuration of the girder and endanger the structural safety. The closure error was approximately 17 cm when the Pasco–Kennewick cable-stayed bridge in Washington was completed because the construction errors were neglected. When the bridge was completed, it did not meet the ideal state and design requirements [1]. Thus, the parameter errors should be adjusted and controlled to ensure that the bridge can meet the design requirements and ideal state. More attention should be given to the construction monitoring of long-span cable-stayed bridges.

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Some methods, such as Kalman's theory, gray prediction theory model, and least squares method, have been used to identify and predict parameter errors in the construction monitoring of bridge structures. Li Yan-pei and Zhao Yinru successfully applied Kalman's theory based on random process control to the construction monitoring of cable-stayed bridges [2-3]. The gray prediction model considers the characteristics of construction that can control the bridge configuration and cable tension of cable-stayed bridges [1, 4, 5]. Cable tension is a key factor in determining the girder configuration and internal force distribution. When the girder configuration deviates from the ideal state, it can be adjusted by increasing or decreasing the cable force. Shang Xin adopted gray theory to adjust the evaluation of a cable-stayed bridge based on cable safety [6]. Chen Changsong integrated the minimum square method, gray prediction theory model, and influence matrix adjustment method to control the concrete cable-stayed bridge to achieve the optimum mechanical state [7]. When more complicated parameters are considered in the construction of bridges, the backpropagation neural network method is capable of computing an extensive range of parameters. Zhou Jiagang and Li Qiao attempted to use the backpropagation neural network method on cable-stayed bridge construction control and obtained a good agreement with the design requirements [8, 9].

In the construction process, the mechanical behaviors are changed by the parameter errors. These errors show strong uncertainty, that is, gray characteristics. In this study, the gray prediction model is applied to analyze the parameter errors and predict deck elevation and initial cable tension in the cable-stayed bridge construction process.

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PRINCIPLES OF THE GRAY PREDICTION MODEL

Gray system theory, an analytical system based on gray relation, processes and manages the original series of data through gray processing and gray generation by using the existing information and the original data [10]. Gray differential equations are established as the main body of the gray prediction model GM(1,1), which can predict future changes in the system. This predictive control method allows the application of gray theory in bridge construction control technology. The method regards the gray dynamic model GM(1,1) as a prediction model. Subsequently, the method is optimized and rectified accordingly. All of the factors that affect the status of bridge construction are eliminated by gray theory and reflected in the target vector concentration, thereby allowing the completion of the prediction process by using the target vector.

The predictive control method predicts the structure state before and after each construction stage, with full consideration of the various factors that affect the construction process and the construction goal for each stage to ensure that the construction proceeds as intended. An error inevitably occurs between the predicted and actual states. Thus, specific errors that affect the construction goal are considered in the subsequent stage. The aforementioned rules are followed until the construction is finished and the structure state fulfills the design.

The construction of long-span bridges is a multivariable, high-level, time-varying, and complex process. Although building an accurate model through this process is difficult, predictive control is an effective program to solve the difficulties in building the model and controlling the system's complexity. In this study, gray system theory is adopted to predict and analyze the construction process of a cable-stayed bridge.

The cable-stayed bridge and its various loads are regarded as a system. Although considerable information is available about the system, further information remains unknown. Therefore, this system is a gray technology system that is affected by noise and has a physical prototype.

In gray system theory, a dynamic model with a group of differential equations is developed, which is called gray differential model GM(1,1). The gray differential model GM(1,1) is a prediction function model with a differential equation expressed as follows:

$$\frac{\mathrm{d}x^{(1)}}{\mathrm{d}t}+a\left(x\right)^{0}=b, \tag{1}$$

Where $x^{(1)}$ is a cumulative value of the raw data series $x^{(0)}$ and calculated as follows:

$$x^{(1)}=\sum_{k=1}^{n}x^{(0)}\left(k\right). \tag{2}$$

Thus, the prediction model is generated by using the data series $x^{(1)}$. The accuracy of the model can be tested by residual size, posteriori, and rolling inspection. If the accuracy of the model cannot meet the requirements, then the model is rectified by the residuals. Following this, feedback correction of the model is implemented through model metabolism.

Only a few data (as few as four) are needed to distinguish and build a GM model. The equation

$$x^{(0)}\left(k\right)+az^{(1)}\left(k\right)=b, k=1,2,\cdots,n \tag{3}$$

is a gray differential model, called GM(1,1), as it includes only one variable $x^{(0)}$, where

$$z^{(1)}\left(k\right)=0.5x^{(1)}\left(k\right)+0.5x^{(1)}\left(k-1\right), k=1,2,\cdots,n. \tag{4}$$

a and b are the coefficients. In the gray system theory , a denotes a developing coefficient and b denotes the gray input. $x^{(0)}\left(k\right)$ is a gray derivative that maximizes the information density of a given series to be modeled.

Based on the least squares method, the parameter $\hat{a}$ is defined as follows:

$$\hat{a}=\left[a\ \ b\right]^{T}=\left(B^{T}B\right)^{-1}B^{T}y_{N},$$

Where

$$B=\begin{bmatrix}-z^{(1)}(2) & 1\\ \cdots & \cdots\\ -z^{(1)}(n) & 1\end{bmatrix}=\begin{bmatrix}-0.5x^{(1)}(1)-0.5x^{(1)}(2) & 1\\ \cdots & \cdots\\ -0.5x^{(1)}(n-1)-0.5x^{(1)}(n) & 1\end{bmatrix},$$

$$y_{n}=\begin{bmatrix}x^{(0)}(2)\\ \cdots\\ x^{(0)}(n)\end{bmatrix},$$

and B is a data matrix. Based on the previously presented equation, the response equations for GM(1,1) are expressed as follows:

$$\hat{x}^{(1)}(k+1)=\left(x^{(0)}(1)-\frac{b}{a}\right)e^{-ak}+\frac{b}{a}, \tag{5}$$

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k), \tag{6}$$

where $\hat{x}^{(1)}(k)$ and $\hat{x}^{(0)}(k)$ denote the calculated values of $x^{(1)}$ and $x^{(0)}$, respectively, at point k. The model GM(1,1) plays an important role in gray forecasting, gray programming and gray control.

CONSTRUCTION PROCESS OF THE CABLE-STAYED BRIDGE

Upon completion of the piers, pylons, and other associated works in the construction process, steel girder erection and cable installation become typical repetitive cycles, as shown in Fig. (**1**). The lifting crane, which is mounted on the last erected segment, hooks up the segment from the transportation barge. This segment is lifted to the design level and is secured to the temporary support frame as a temporary fixing. This segment is welded to the preceding segment. Following this, the inclined cable is installed on the segment. The cable is first anchored to the tower and the deck. After the anchorage work is completed, the cable is stressed by a heavy jack until the designed length is achieved. One back-span cable and one mid-span cable should be stressed simultaneously to balance the bending

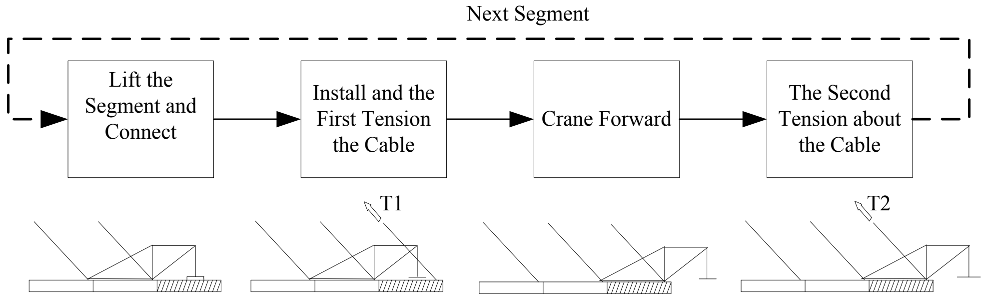


Fig. (1). Construction process of a standard girder.

force acting on the tower. The typical erection cycle is repeated until the last closure segment, which is operated with a distinct method.

CONSTRUCTION MONITORING OF CABLE-STAYED BRIDGES

For cable-stayed bridges, when various factors are effectively controlled during the construction process, the alignment and internal force of the girder can be adjusted only by initializing the heights of girders and the tension forces of cables. Therefore, the initial heights of girders and the tension forces of cables are two control inputs of the system. Given that a change in initial girder height does not affect the internal structural force, the alignment can be adjusted using the initial height without changing the internal structural force. As a result, the completed stage alignment need not be changed for the design alignment target. Internal structural forces are generated by the difference between various parameters of construction and the design values. Only the tension forces of cables are adjusted to allow the internal force distribution to meet the design expectations. Therefore, the ultimate goal of the control method is to adjust the girder alignment in the completed stage and ensure that the distribution of internal structural forces meets the design requirements.

(1) Adjustment of Cable Tension

The increment of vertical displacement of the cantilever end girder and several adjacent segments during construction should be consistent with the design value by confirming the initial tension forces of cables in the succeeding phase, in which $T_r(k+1)$ segment is tensioned and $(k+1)$ segment is welded. The cable to be tensioned at the cantilever end is denoted as the first i number, and the corresponding cantilever end node is the i node. The girder segment constructed before the i number cable is tensioned and denoted as the i number segment. The initial tension force of the cable is $T_r(k+1)$ when the $(k+1)$ segment is welded, and it is expressed as follows:

$$T_r(k+1) = \beta(k+1) \cdot T_d(k+1), \tag{7}$$

Where $\beta(k+1)$ is the tension adjustment factor of the $(k+1)$ number cable.

(2) Confirmation of Girder Elevation

① Prediction of the vertical displacement increment at the cantilever end before and after the cable is tensioned

The adjustment value of the $(k+1)$ number cable tension force is determined based on the definition series x_1. The design value is $T_d(k+1)$, and the vertical displacement increment design value is $\Delta_{1d}(k+1)$. Thus, the corresponding model output value $\Delta_{1m}(k+1)$ is expressed as follows:

$$\Delta_{1m}(k+1) = T_r(k+1) \cdot \frac{\Delta_{1d}(k+1)}{T_d(k+1)} x_1(k+1). \tag{8}$$

② Prediction of the Vertical Displacement Increment at the Cantilever end Before and After the Girder Segment is Lifted

Based on the definition series y_1, the vertical displacement increment of the design value for the cantilever

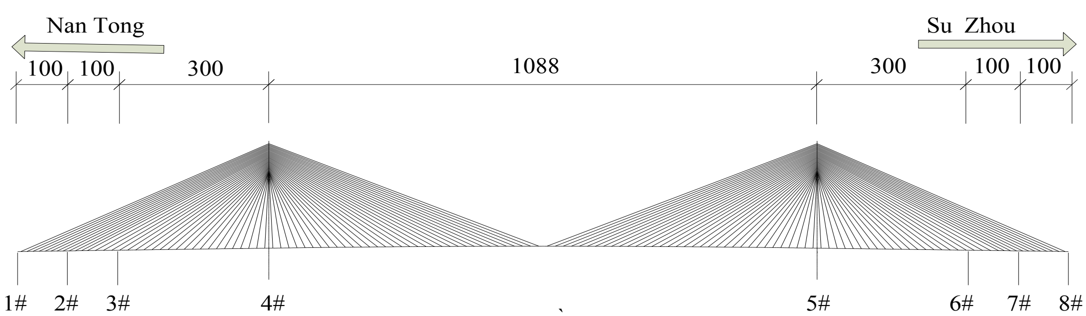


Fig. (2). Layout of the Sutong cable-stayed bridge.

Table 1. Measured and predicted values of $x_1(i)$.

No.	Cable No.	Measured Displacement (mm)/Measured Cable Tension (kN)		Theoretical Displacement (mm)/Design Cable Tension (kN)	$x_1(i)$	
		Side Span	Main Span		Side Span	Main Span
1	6	40/4,508	47/4,530	56/4,466	0.708	0.827
2	7	78/4,821	82/4,850	96/4,741	0.799	0.835
3	8	132/5,120	137/5,132	148/5,046	0.879	0.910
4	9	187/5,530	192/5,542	213/5,436	0.863	0.884
5	10			282/5,840	(0.835)	(0.884)

end is $\Delta'_{1d}(k+1)$ when the $(k+1)$ number girder segment is poured. Thus, the corresponding model output value $\Delta'_{1m}(k+1)$ is expressed as follows:

$$\Delta'_{1m}(k+1) = y_1(k+1)\cdot\Delta'_{1d}(k+1). \tag{9}$$

③ Prediction of the Hanging Basket Deformation when Lifting the Girder Segment

If girder elevation is determined during construction, then deformation of the crane must be predicted accordingly. $z_1(i)$ is defined as the ratio between the actual deformation value of the hanging basket and the theoretical value when the i number girder segment is lifted. The prediction model GM(1,1) is set up for $z_1(i)$, and the crane deformation value is $\Delta''_{1d}(k+1)$. As a result, the model output value is expressed as follows:

$$\Delta''_{1m}(k+1) = z_1(k+1)\cdot\Delta''_{1d}(k+1). \tag{10}$$

④ Confirmation of Segment Elevation at $(k+1)$ Stage $H_l(k+1)$

Girder elevation at the $(k+1)$ number stage is calculated as follows:

$$\begin{aligned} H_1(k+1) &= H_d(k+1) + \Delta'_{1m}(k+1) \\ &\quad + \Delta''_{1m}(k+1) - \Delta_{1m}(k+1), \end{aligned} \tag{11}$$

Where $H_d(k+1)$ is the design elevation value of the cantilever end after the $(k+1)$ number cable is tensioned. Consequently, once the cable tension is determined at the $(k+1)$ stage, the predicted values of $\Delta_{1m}(k+1)$, $\Delta'_{1m}(k+1)$, and $\Delta''_{1m}(k+1)$ can be obtained from the past and present data. Subsequently, the unique elevation value can be identified.

APPLICATION OF GM(1,1) MODEL TO THE CONSTRUCTION MONITORING OF CABLE-STAYED BRIDGES

(1) Project Overview of Sutong Cable-Stayed Bridge

The Sutong Bridge crosses the Yangtze River approximately 100 km upstream from Shanghai, China and connects the cities of Suzhou and Nantong located at the southern and northern banks, respectively. The bridge is a seven-span double-pylon and double-cable plane steel box girder cable-stayed bridge, with a span arrangement of 100 + 100 + 300 + 1,088 + 300 + 100 + 100 = 2,088 m, as shown in Fig. (**2**). The Sutong Bridge is the second longest cable-stayed bridge in the world, with a record-breaking construction in the history of bridge building.

Table 2. Measured and predicted values of $y_1(i)$.

No.	Girder Segment No.	Measured Displacement (mm)		Theoretical Displacement (mm)	$y_1(i)$	
		Side Span	Main Span		Side Span	Main Span
1	6	65	70	83	0.783	0.843
2	7	102	110	138	0.739	0.797
3	8	166	180	210	0.790	0.857
4	9	230	248	293	0.785	0.846
5	10			390	(0.783)	(0.821)

(2) Engineering Application

Gray theory is applied to the construction control of the cable-stayed bridge. The construction state is as follows. No. 15 cable is tensioned, and the initial height of the No. 16 segment is predicted. At the same time, the initial tension force of the No. 16 cable is determined.

1) Confirmation of $x_1(i)$, the Vertical Displacement Increment Prediction for the Cantilever End When the No. 10 Cable is Tensioned

The measured series $x_1(i)$ identifies four cables (Nos. 6, 7, 8, and 9) that have been tensioned, and the No. 10 cable, which will be tensioned, as a group. The model GM(1,1) is established based on the actual and theoretical data. The prediction results are shown in Table **1**.

2) Confirmation of $y_1(i)$, the Vertical Displacement Increment Prediction for the Cantilever end when the No. 10 Girder Segment is Erected

The measured series $y_1(i)$ identifies the adjacent four girder segments, which have been constructed. The model GM(1,1) is established based on the actual and theoretical data. The prediction results are shown in Table **2**.

3) Confirmation of $z_1(i)$, Prediction of Crane Deformation

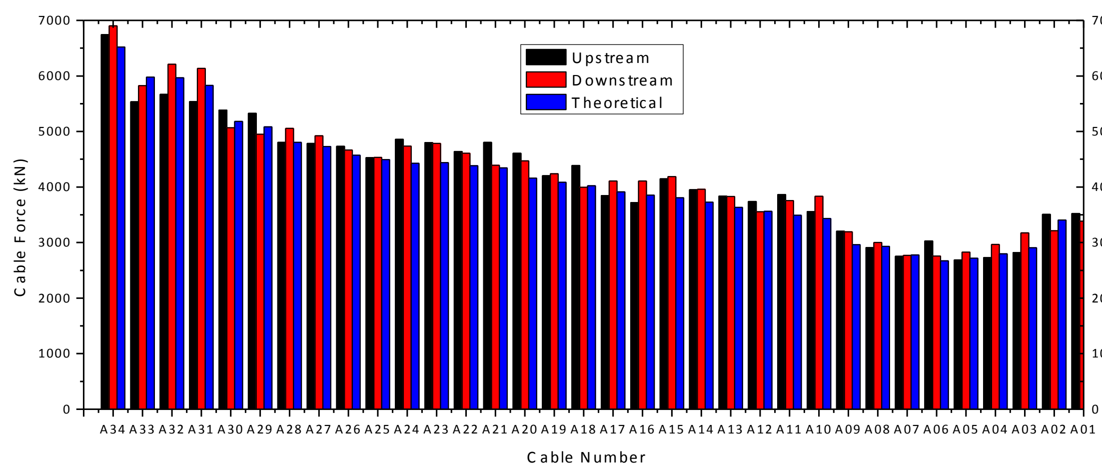
The theoretical crane deformation is 80 mm. From the measured data, the prediction results of crane deformation are as follows: 1.36 for side span $z_1(5)$ and 1.17 for the main span $z_1(5)$.

4) Prediction of the Initial Cable Tension for the No. 10 Cable

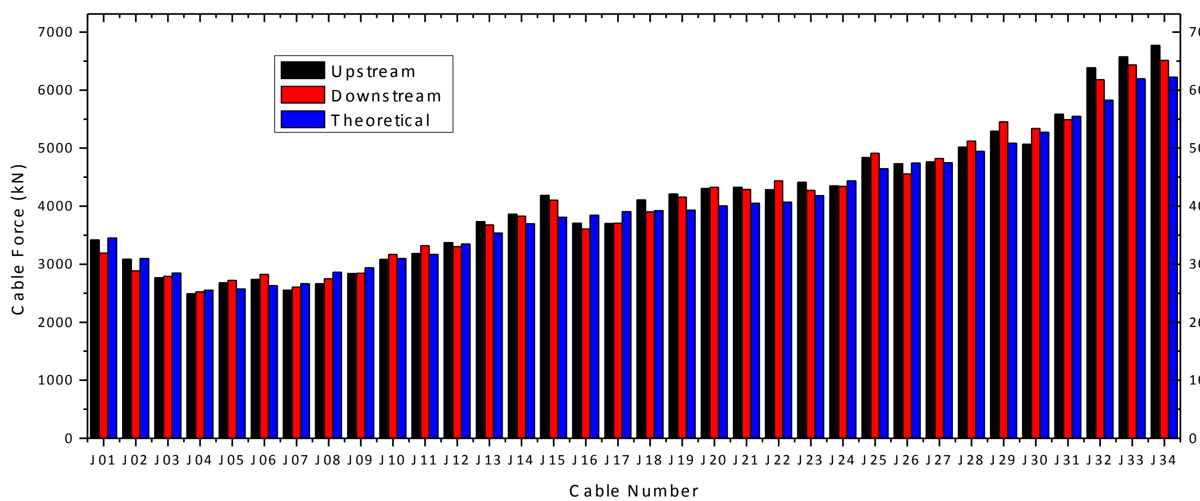
Based on the determined value of $x_1(5)$ and the predicted value of $y_1(5)$, the adjustment factor $\beta_1(10)$ of the tension force of the No. 10 cable and the corresponding cable force are obtained as follows:

No. 10 cable of the side span: $\beta_1(10) = 1.026$; $T_r(10) = 1.026 T_d(10) = 5{,}992$ kN,

No. 10 cable of the main span: $\beta_1(10) = 1.014$;



(a) Side span cable forces.



(b) Main span cable forces.

Fig. (3). Cable tension at the completed stage (kN).

$T_r(10) = 1.014 T_d(10) = 5{,}922$ kN.

(3) Predicted Value of the Initial Evaluation of the No. 10 Girder Segment

Prior to building the No. 10 girder segment, the initial evaluation of the side span and main span is first analyzed. Subsequently, the predicted initial height is compared with the theoretical height.

First, the result $T_r(10) = 5{,}992$ kN obtained from the previously presented equation is substituted into formula (8). Thus, the predicted value of the vertical displacement increment of the cantilever end $\Delta_{1m}(10)$ is obtained as follows:

$$\Delta_{1m}(10) = T_r(10) \cdot \frac{\Delta_{1d}(10)}{T_d(10)} x_1(10)$$
$$= 1.026 \times 0.835 \times 282 = 242 \text{ mm.}$$

Second, when the No. 10 segment is set up, the predicted value of the vertical displacement $\Delta'_{1m}(10)$ can be obtained by using formula (9), as follows:

$$\Delta'_{1m}(10) = y_1(5)\Delta'_{1d}(10) = 0.783 \times 390 = 305 \text{ mm.}$$

Finally, when the No. 10 segment is set up, the predicted value of the hanging basket deformation may be obtained by using formula (10), as follows:

$$\Delta''_{1m}(10) = z_1(5)\Delta''_{1d}(10) = 1.36 \times 80 = 109 \text{ mm.}$$

The design evaluation value of the girder bottom is $H_d(10) = 73.204$ m after the No. 10 cable is tensioned. The results are substituted into formula (11). Therefore, the predicted value of initial height $H_d(10)$ is calculated as follows:

$$H_1(10) = H_d(10) + \Delta'_{1m}(10) + \Delta''_{1m}(10) - \Delta_{1m}(10) = 73.376 \text{ m.}$$

The theoretical value of the No. 10 girder segment was observed to be 73.402 m, whereas the actual value should be lower by 2.6 cm.

As the No. 10 segment of the main span was erected, the predicted value of the initial evaluation $H_1(10)$ was 77.727 m, which was calculated using the aforementioned method. The theoretical value of the initial evaluation of this segment was 77.698 m. Consequently, the actual initial height should be higher by 2.9 cm.

When the gray system theory was used in the construction control of the cable-stayed bridge, positive results were obtained during construction. Generally, the deviation error of the cable force during the completed stage was not greater than 10% and the closure error was only 5 mm.

The gray prediction model GM(1,1) was applied to construction monitoring of the Sutong cable-stayed bridge. This model can control the parameters and predict the errors in the subsequent stage effectively. When the structure was completed, the cable force distribution was observed to be smooth and did not involve non-mutation. Furthermore, the tensions in the cables were in an ideal state. The actual configuration of the girder was in good agreement with the theoretical alignment, as shown in Fig. (**4**). The upstream and downstream cable forces were measured in the field after the Sutong cable-stayed bridge was completed [11]. A comparison of the calculated and measured results shows that a slight difference existed between them, as shown in Fig. (**3**). The deviation error of the cable force during the completion stage was not greater than 10%. Furthermore, cable tension stress was in the range of Chinese design standards. The deformation of the girder and the tower is sensitive to the cable force. During construction, the cable forces are usually adjusted to change the internal force as well as the displacement of the girder and the tower.

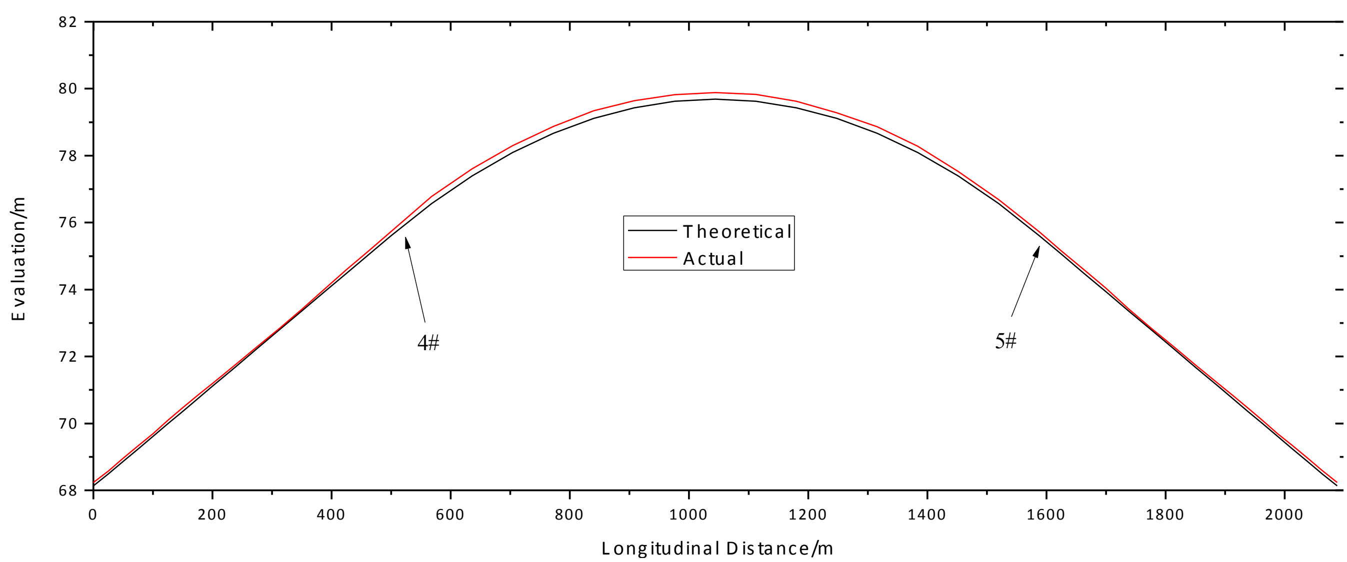


Fig. (4). Theoretical and actual evaluation at the completed stage.

Furthermore, the cable force is a key factor in determining the actual geometric profile of the completed stage.

CONCLUSION

The internal relation between construction monitoring data and the gray prediction model GM(1,1) was investigated in this study. The model GM(1,1) was set up based on the bridge characteristics, which were applied to predict the deformation of the girder and the cable tension in the construction monitoring of the Sutong cable-stayed bridge in China. The parameters of the construction process were successfully predicted, and good results were achieved. Based on the previously presented analysis, the following conclusions can be drawn:

(1) The construction process of the cable-stayed bridge was regarded as a gray process and was investigated by gray system theory. Although randomness weakened the data, gray system theory highlighted the regularity of the data generated. Several random variables during the construction of the cable-stayed bridge were successfully predicted using the model GM(1,1).

(2) Initial cable tension and initial evaluation of the cable-stayed bridge constructed via the cantilever method were regarded as two control inputs, which enabled prediction of the subsequent stage. At the same time, the gray prediction model GM(1,1) was rectified accordingly based on specific circumstances in the construction site. Finally, construction control of the cable-stayed bridge achieved good prediction precision.

CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

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